Fiscal Deficits, Current Account Dynamics and Monetary Policy

Giorgio Di Giorgio and Salvatore Nisticò

November 2007

Outputs from LLEE research in progress, as well contributions from external scholars and draft reports based on LLEE conferences and lectures, are published under this series. Comments are welcome. Unless otherwise indicated, the views expressed are attributable only to the author(s), not to LLEE nor to any institutions of affiliation.

© Copyright 2007, Giorgio Di Giorgio and Salvatore Nisticò
Freely available for downloading at the LLEE website (http://www.luiss.it/ricerca/centri/llee)
Email: llee@luiss.it
Fiscal Deficits, Current Account Dynamics and Monetary Policy*

Giorgio Di Giorgio° and Salvatore Nisticò§

LLEE Working Document No.55 November 2007

Abstract

This paper develops a tractable stochastic two-country “perpetual youth” Dynamic New Keynesian model of the international business cycle with incomplete international financial markets and stationary net foreign assets. The model allows for a thorough analysis of the interactions of monetary policy with endogenous, non-balanced budget fiscal policy. We use the model to derive the dynamic and cyclical properties of fiscal deficit feedback rules, recently in the spotlight of the empirical literature. We also study a wide range of structural shocks under alternative monetary and fiscal policy regimes, and discuss the implications for net foreign assets and exchange rate dynamics. Our results imply that a crucial role for the dynamics of net foreign assets is played by the degree of “fiscal discipline”, i.e. the extent to which the fiscal rule responds to debt dynamics. We show that under low fiscal discipline (characterizing most industrialized countries, first and foremost the U.S.) temporary positive productivity shocks may result in highly persistent deteriorations of the external position. Our results also suggest that any attempt by monetary policy alone to stabilize the dynamics of net foreign assets would induce excessive and costly fluctuations of the exchange rate.

JEL classification: E43, E44, E52, E58
Key words: Twin Deficits, Net Foreign Assets, DSGE Models, Monetary and Fiscal Policy.

(*) The authors wish to thank Guido Ascari, Hafedh Bouakez, Efrem Castelnuovo, Michael Kumhof, Anna Lipinska, Francesco Lippi, Luca Sessa, and participants at the 13th International Conference on Computing in Economics and Finance (HEC, Montréal), the 2nd Meeting on Open Macroeconomics and Development (CEDERS, Aix-en-Provence) and the 2nd PIERO MONCASCA Money-Macro Workshop (Padova). Usual disclaimers apply.

(°) Libera Università Internazionale degli Studi Sociali (LUISS) “Guido Carli”, Viale Romania 32, 00197 Rome, Italy. Email: gdg@luiss.it

(§) Università di Roma “Tor Vergata” and Libera Università Internazionale degli Studi Sociali (LUISS) “Guido Carli”, Viale Romania 32, 00197 Rome, Italy. Email: snistico@luiss.it
1 Introduction

A renewed interest in the dynamics of the current account spurred by productivity and fiscal shocks has recently emerged as a consequence of the record deterioration of the current account balance in the US, which has been accompanied first by the faster productivity growth of the 90’s, and then by the new large fiscal deficits run in the aftermath of September 11.

Standard economic theory has a simple textbook argument to explain the negative effect of large fiscal deficits on the current account: an increase in public spending or a reduction in taxes lowers national savings; if investments do not react too much (as supported by most empirical evidence), it is necessary for the trade balance to match the reduction in national savings and for the country to become a net debtor. The effect on the current account will depend upon the degree of capital mobility (and substitutability) among foreign and domestic assets, and on the degree of rationality assumed: agents may in fact react to fiscal deficits today by expecting future surpluses and may be willing to increase private savings today in order to be able to pay higher future taxes and smooth consumption intertemporally (ricardian equivalence hypothesis).

A powerful tool to integrate such long-run view with fully specified short-run dynamics, may be provided by open economy Dynamic Stochastic General Equilibrium (DSGE) models.

The New Open Economy Macroeconomics (NOEM) literature provides several contributions analyzing virtually all kinds of monetary policies within a DSGE framework. However, in the same literature the analysis of fiscal behavior has been given considerably less attention thus far. Moreover, it has been mainly limited to the analysis of balanced budget (BB) policies. The benchmark open economy model in the NOEM tradition (the Redux model of Obstfeld and Rogoff, JPE 1995) builds on the joint assumption of infinitely lived household and frictionless financial markets. Hence, this model results in non-stationary net foreign asset dynamics; moreover ricardian equivalence in this setting severely limits the range of fiscal policies that can be studied. On the contrary, the Stability and Growth Pact in the European Union stimulated an increasing number of empirical studies focusing on discretionary fiscal policy as a stabilization tool. Such studies emphasized the empirical performance of different endogenous fiscal deficit feedback rules.

2007). Here we extend the framework to allow for endogenous determination of current account dynamics and fiscal deficits.

A linear approximation of the model is used to derive the cyclical and dynamic properties of fiscal-deficit feedback rules, and to study their interaction with monetary policy following a wide range of structural shocks.

The demand-side of our economy is a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (JPE, 1985) and Yaari (RES, 1965). Consumers, in each country, supply labor services in a domestic competitive labor market and demand consumption goods and financial assets issued in the two currencies. In addition, each consumer in each country is endowed with an equal amount of non-tradeable shares of the domestic firms. We assume the following financial structure: markets are complete within each country and incomplete internationally. The only assets tradable across borders are one-period zero-coupon bonds issued in the two currencies by the governments to finance their budget deficits. The joint assumption of non-ricardian consumers and incomplete international financial markets is key to generate determinacy of the steady state stock of net foreign assets, and allows for a thorough analysis of current account dynamics in a DSGE framework. Moreover, the assumption of non-ricardian agents also allows to study a wider range of fiscal shocks compared to the Representative Agent (RA) setup.

The supply-side is standard and consists of a continuum of imperfectly competitive firms operating in each country. These firms hire labor from domestic households to produce differentiated consumption goods. Firms set their prices according to a Calvo rule that allows only a fraction of them to optimally revise prices in a given period in order to maximize profits.

The government in each country purchases a fraction of domestically produced consumption goods, raises lump sum taxes and issues riskless nominal bonds to finance its budget deficit. We assume governments to make fiscal policy by controlling the level of real taxes, given an exogenous process for government expenditures, and compare alternative fiscal regimes defined in terms of specific targets for the primary budget deficit. In particular, we focus on the specification used in some recent empirical studies (such as Gali and Perotti, EP 2003), that assumes the primary fiscal deficit to be set according to a counter-cyclical feedback rule reacting to the output gap and the existing stock of outstanding debt.

Monetary policy is modeled by means of usual instrument rules à la Taylor. We consider different policy regimes and compare them with a Wicksellian Policy that implements the flexible-price allocation. Our feedback rules have interest rates responding to either domestic inflation or CPI inflation. We also consider the effect of an exchange rate peg and the case in which the Central Bank aims at stabilizing NFA dynamics by means of an aggressive managed floating.
We study the impact of relative fiscal and productivity shocks on external and fiscal imbalances and the dynamics of the exchange rate by simulating the model under a set of different combinations of monetary and fiscal rules.

Our main results can be summarized as follows. 1) Under endogenous counter-cyclical fiscal policy, the degree of fiscal discipline plays a crucial role for the NFA and exchange rate dynamics. We define the degree of fiscal discipline as the degree of responsiveness of the deficit rule to the stock of outstanding debt. 2) Being consumption smoothing the only motive for foreign asset accumulation, under BB fiscal policy a positive productivity shock in our framework can only produce a current account surplus. However, if the government follows a counter-cyclical deficit rule with scarce fiscal discipline (as the empirical evidence for the U.S. suggests), then positive productivity shocks may result in persistent deteriorations of the external balance in the medium run. 3) Under scarce fiscal discipline, persistent negative imbalances may result also from purely temporary monetary restrictions. 4) Any attempt by monetary policy alone to stabilize the dynamics of NFA would imply excessive volatility of the exchange rate, inflation and output. 5) On the contrary, endogenous fiscal policy characterized by high fiscal discipline can prevent persistent external imbalances from arising, regardless of the monetary policy rule adopted by the Central Bank. 6) Expansionary, country-specific fiscal shocks unambiguously appreciate the exchange rate on impact and depreciate it in the long-run, regardless of the nature of the shock (unlike in the Redux model and in Ganelli, JIE 2005).

The paper is organized as follows. After a brief subsection dealing with the related literature, in section 2 we lay out the model. In section 3 we describe the linearized version of the model and the behavior of monetary and fiscal authorities. Section 4 presents the implications of different policy scenarios for the dynamics of the current account and the other main endogenous variables. Section 5, finally, summarizes and concludes.

1.1 Related Literature

This paper offers a methodological contribution to the NOEM literature based on DSGE models. As mentioned earlier, this literature, moving from the seminal contribution by Obstfeld and Rogoff (JPE, 1995), has to face a relevant issue regarding stationarity of NFA: in the Redux model, the joint assumption of Ricardian Agents and frictionless financial markets implies indeterminacy of the steady-state holdings of net foreign assets.

Up to now, the existing literature has taken two different routes to overcome this limit. Benigno (CEPR, 2001), Erceg–Guerrieri–Gust (IF, 2005) and Hunt–Rebucci (IF, 2005) among others, chose to retain the fully stochastic RA structure of the demand side. However, they introduced additional frictions (in the form of intermediation costs) in the international financial markets, in order to introduce a link between consumption and NFA dynamics and
thus achieve stationarity. The second approach, adopted by Cavallo–Ghironi (JME, 2002), Smets–Wouters (JME, 2002) and Ganelli (JIE, 2005) among the others, maintained a frictionless financial structure while introducing non-Ricardian agents, so that the evolution of financial wealth affects consumption and stationarity is again achieved. In order to solve the model, however, these contributions take a step back to the Redux model by restoring certainty equivalence. This approach provides a natural environment for the analysis of non-BB fiscal policies. However it does not allow to say much about the cyclical properties of different rules, because of the perfect foresight assumption.

This paper makes a step forward, as it provides a fully stochastic NOEM DGE framework in which consumers are non-ricardian and financial markets are frictionless (though incomplete). In this perspective, here we modify the financial structure of the stochastic non-ricardian open economy model analyzed in Di Giorgio–Nisticò (JMCB, 2007).3

Our results can be compared with those of several related contributions.

Cavallo–Ghironi (JME, 2002) study the dynamic effects of unexpected productivity shocks for exchange rate determination and NFA dynamics, in a perfect foresight framework. They show that, under a monetary policy rule responding to the level of output (rather than the output gap), an unexpected positive productivity shock deteriorates the external balance only if it is permanent, while it always appreciates the exchange rate.

Analogously, Ganelli (JIE, 2005) introduces a perpetual youth structure in the Redux model, to show the implications for exchange rate dynamics of alternative (unexpected) fiscal shocks. In his framework, a tax cut unambiguously drives the exchange rate to appreciate on impact and depreciate in the long run, while the effects of debt-financed government expenditures are ambiguous. We will argue that this ambiguity stems from the assumption that public consumption is uniformly distributed over domestic and foreign goods.

Two recent papers that have tried to explain sustained deterioration of the U.S. external position in the 90’s, in the face of a stronger productivity growth are Hunt–Rebucci (IF, 2005) and Fogli–Perri (2006). Hunt–Rebucci (IF, 2005) simulate the quantitative GEM model of the IMF and show that a number of additional features are still needed to match the observed dynamics, namely uncertainty and learning about the persistence of productivity shocks and a consumers portfolio preference shift in favor of U.S. assets. By using a standard two-country Real Business Cycle model with borrowing limits, Fogli–Perri (2006) argue that the higher relative reduction in the volatility of structural shocks in the U.S. produced a fall in precautionary savings, able to account for about 20% of the observed deterioration in the U.S. external balance.

We develop a tractable stochastic model that is able to address the issues above from a

---

qualitative perspective. In particular, we assess the role of fiscal and monetary policy for the dynamics of the exchange rate and NFA induced by fiscal and productivity shocks. We characterize fiscal policy as implemented by means of endogenous feedback rules, which have the primary fiscal deficit vary in response to some measure of the cycle, like the output gap, and the stock of outstanding debt, following similar specifications used in the empirical literature. Gali–Perotti (EP, 2003) estimate deficit feedback rules for a wide set of industrialized countries with the aim of establishing whether or not the SGP has limited the ability of EMU countries to implement counter-cyclical fiscal policy relative to other non-EMU countries. Their estimates for the U.S. provide strong evidence of a counter-cyclical conduct during the 90’s but not in the 80’s. They did not find any evidence of a significant response to debt dynamics.\(^4\)

We will show that this evidence suggests that fiscal policy may have played a relevant role in driving the observed dynamics of the U.S. external balance.

2 A DSGE Two-Country Model with Incomplete Markets

In this section we present a two-country OLG model which draws on the analysis developed in Di Giorgio and Nisticò (2006). This specification, coupled with the assumption of incomplete international bond markets, generates determinacy of the stock of net foreign assets in the steady state and allows for a thorough analysis of current account dynamics in a DSGE framework.

Following Obstfeld and Rogoff (JPE, 1995) and Benigno and Benigno (REStud, 2003), we model the world economy as consisting of a continuum of households and firms in the interval \([0, 1]\), divided in two countries \(H\) and \(F\), of dimension \(n\) and \((1 - n)\) respectively.

The two countries are structurally symmetric. Each household, in each country, supplies labor inputs to firms and demands a bundle of consumption goods consisting of both home and foreign goods.

The productive sector produces a continuum of perishable goods, which are differentiated across countries and with respect to one another. To assign a relevant role to monetary policy\(^5\) we introduce nominal rigidities by assuming that both domestic and foreign firms, each period, face an exogenous probability of optimally changing the price of their good (see \(^{4}\)Favero–Monacelli (2005) estimate an analogous feedback rule for the U.S. only, adopting Markov-switching regression methods, and find evidence of a switch to a passive (in the terminology of Leeper, JME 1991) and more counter-cyclical regime during the 90’s

\(^5\)One should notice that, although we do not model money holdings for simplicity, we assign to the Central Bank the role of setting the interest rate according to some optimal “rule”. This is the sense by which we talk about monetary policy.
2.1 The Demand-Side.

The demand-side of our economy is a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (1985) and Yaari (1965). Each period a constant-sized cohort of households enters each country, facing a constant probability $\gamma$ of dying before the next period begins. To abstract from population growth the cohort size is set equal to $\gamma$.

2.1.1 Intertemporal Allocation.

Consumers have log-utility preferences over consumption and leisure, supply labor services in a domestic competitive labor market and demand consumption goods. Moreover, they allocate savings among a full set of domestic state contingent private securities and two internationally traded riskless financial assets issued in the two currencies by the governments to finance their budget deficits. Each consumer in each country is endowed with an equal amount of non-tradable shares of the domestic firms.

Let $E_t$ be the nominal exchange rate defined as the domestic price of foreign currency, $T^i_t(j)$ denote real lump-sum taxes levied by the fiscal authority of country $i$ on household $j$, and variables with a superscript $^*$ denote nominal values. In particular, $B^*_{k,t}(j)$ denotes holdings of risk-free assets held by generation $j$ living in country $i$, in nominal terms and denominated in the currency of country $k$, for $i, k = H, F$: such financial assets are one-period zero-coupon bonds issued by the governments. $Q^*_{i,t}(j)$ denotes cohort $j$’s holdings of the portfolio of state-contingent assets, denominated in domestic currency, for which the relevant discount factor pricing one-period claims is $F^i_{t,t+1}$. Moreover, given the law of one price and the absence of any home bias in consumption (see the section on intratemporal allocation), it follows that $P^H_t = E_t P^F_t$. Each household enters each period with a stock of bond holdings which also pays off the return on the insurance contract à la Blanchard (1985).

Then, the optimization problem faced at time 0 by the representative consumer of cohort $j < 0$ living in country $i$ is to choose $\{C^i_t(j), N^i_t(j), Q^*_{i,t}(j), B^*_{H,t}(j), B^*_{F,t}(j)\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (1 - \gamma)^t \left[ \log C^i_t(j) + \delta \log (1 - N^i_t(j)) \right]$$

---

6 We interpret the concepts of “living” and “dying” in the sense of being or not being operative in the markets and thereby affecting or not economic activity through decision-making processes. In this perspective, the expected life-time $1/\gamma$ is interpreted as the effective decision horizon. See also Leith and Wren-Lewis (2000), Leith and von Thadden (2004), Nisticò (2005) and Piergallini (2004).

7 The stochastic discount factor is unique, within each country, given the assumption of complete domestic markets.
subject to a sequence of budget constraints of the form

\[
P^H_t C^H_t(j) + E_t\{F^H_{t,t+1}Q^H_{H,t}(j)\} + B^H_t(j) + E_t B_t^H(j) \leq \frac{1}{1-\gamma} \left[ (1 + r^H_{t-1}) B^H_{t-1}(j) + E_t(1 + r^F_{t-1}) B^H_{t-1}(j) + Q^H_{H,t-1}(j) \right] + \\
+ W^H_t N^H_t(j) + P^H_t D^H_t(j) - P^H_t T^H_t(j) \quad (1)
\]

for households living in country \( H \), in which \( D^H_t(j) \equiv \frac{1}{n^H_t} \int_0^n D^*_H(t, j) \, dh \) denotes \( j \)'s claims on real profits from domestic firms, and

\[
P^F_t C^F_t(j) + E_t\{F^F_{t,t+1}Q^F_{F,t}(j)\} + B^F_t(j) + E_t B_t^F(j) \leq \frac{1}{1-\gamma} \left[ (1 + r^H_{t-1}) B^F_{H,t-1}(j) + (1 + r^F_{t-1}) B^F_{F,t-1}(j) + Q^F_{F,t-1}(j) \right] + \\
+ W^F_t N^F_t(j) + P^F_t D^F_t(j) - P^F_t T^F_t(j) \quad (2)
\]

for those living in country \( F \), in which \( D^F_t(j) \equiv \frac{1}{(1-n)^F_t} \int_n^1 D^*_F(t, f) \, df \).

Let \( \Lambda^i_t(j) \) denote the lagrange multiplier on the constraint for household \( j \) living in country \( i \). The FOCS are:

\[
\mathcal{F}^i_{t,t+1} = \beta \frac{\Lambda^i_{t+1}(j)}{\Lambda^i_t(j)} \quad (3)
\]

\[
\Lambda^i_t(j) P^i_t C^i_t(j) = 1 \quad (4)
\]

\[
\Lambda^i_t(j) W^i_t(1 - N^i_t(j)) = \delta \quad (5)
\]

\[
\delta P^i_t C^i_t(j) = W^i_t(1 - N^i_t(j)) \quad (6)
\]

\[
\Lambda^H_t(j) = \beta (1 + r^H_t) E_t \Lambda^H_{t+1}(j) \quad (7)
\]

\[
\mathcal{E}^i_t \Lambda^H_t(j) = \beta (1 + r^F_t) E_t \mathcal{E}^i_{t+1} \Lambda^H_{t+1}(j) \quad (8)
\]

\[
\Lambda^F_t(j) = \beta (1 + r^F_t) E_t \Lambda^F_{t+1}(j) \quad (9)
\]

\[
\Lambda^F_t(j) = \beta (1 + r^H_t) E_t \mathcal{E}^i_t \Lambda^F_{t+1}(j) \quad (10)
\]

Given the condition \( \mathcal{F}^i_{t,t+1} = \beta \frac{\Lambda^i_{t+1}(j)}{\Lambda^i_t(j)} \), for all \( i = H, F \), we derive the two UIP conditions:

\[
E_t \left\{ \mathcal{F}^H_{t,t+1} \left[ \frac{\mathcal{E}^i_{t+1}}{\mathcal{E}^i_t} (1 + r^F_t) - (1 + r^H_t) \right] \right\} = 0 \quad (11)
\]

\[
E_t \left\{ \mathcal{F}^F_{t,t+1} \left[ \frac{\mathcal{E}^i_{t+1}}{\mathcal{E}^i_t} (1 + r^H_t) - (1 + r^F_t) \right] \right\} = 0 \quad (12)
\]
We can also define nominal financial wealth carried over from period \( t - 1 \) as:

\[
\Omega_{t-1}^F(j) \equiv \frac{1}{1 - \gamma} \left[ (1 + r_{t-1}^H) B_{H,t-1}^*(j) + E_t(1 + r_{t-1}^F) B_{F,t-1}^*(j) + Q_{H,t-1}^* (j) \right]
\]

(13)

\[
\Omega_{t-1}^F(j) \equiv \frac{1}{1 - \gamma} \left[ (1 + r_{t-1}^H) B_{H,t-1}^*(j) + (1 + r_{t-1}^F) B_{F,t-1}^*(j) + Q_{F,t-1}^* (j) \right].
\]

(14)

As a consequence, equilibrium conditions in the asset markets imply

\[
E_t F_{t,t+1}^H (1 - \gamma) \Omega_t^H(j) = B_{H,t}^*(j) + E_t B_{F,t}^H(j) + E_t \{ F_{t,t+1}^H(j) \}
\]

(15)

\[
E_t F_{t,t+1}^F (1 - \gamma) \Omega_t^F(j) = B_{F,t}^*(j) + E_t B_{F,t}^F(j) + E_t \{ F_{t,t+1}^F(j) \}.
\]

(16)

Accordingly, we can re-write the budget constraint for generic cohort \( j \) living in country \( i \) in period \( t \) as a stochastic difference equation in the nominal financial wealth:

\[
P_t^i C_t^i(j) + E_t F_{t,t+1}^F (1 - \gamma) \Omega_t^F(j) = \Omega_{t-1}^F(j) + P_t^i \omega_t^i(j),
\]

(17)

in which \( \omega_t^i(j) \equiv \frac{w_t^i}{P_t^i} N_t^i(j) + D_t^i(j) - T_t^i(j) \) denotes real non-tradable human and financial wealth. The above equation can be solved forward to express personal nominal consumption as a linear function of total tradable and non-tradable wealth:

\[
P_t^i C_t^i(j) = \zeta (\Omega_{t-1}^F(j) + \mathcal{H}_t^i(j)),
\]

(18)

where \( \zeta \equiv 1 - \beta (1 - \gamma) \) is the propensity to consume out of total wealth, which is common across cohorts and over time, and \( \mathcal{H}_t^i(j) \) is the expected discounted stream of future nominal non-tradable wealth:

\[
\mathcal{H}_t^i(j) \equiv E_t \left\{ \sum_{k=0}^{\infty} F_{t,t+k}^i (1 - \gamma)^k P_{t+k}^i \omega_{t+k}^i(j) \right\}.
\]

(19)

The governments finance their nominal budget deficits by issuing nominal debt denominated in local currency. This implies, in per-capita terms:

\[
B_{i,t}^* = (1 + r_{t-1}^i) B_{i,t-1}^* + Z_{i,t}^*;
\]

(20)

where \( Z_{i,t}^* \) denotes the nominal primary deficit for country \( i \), defined as

\[
Z_{i,t}^* \equiv P_{i,t}^i G_t^i - P_{i,t}^i T_t^i.
\]

(21)

Since all variables in equation (20) are in per-capita terms, and the public debt of each government can be bought by residents in either one of the two countries, the following must hold:

\[
B_{H,t}^* \equiv \frac{n B_{H,t}^{HF} + (1-n) B_{F,t}^{HF}}{n} \]

(22)

\[
B_{F,t}^* \equiv \frac{n B_{F,t}^{HF} + (1-n) B_{F,t}^{HF}}{1 - n}.
\]

(23)
Aggregating across cohorts, it is possible to write the relevant equilibrium conditions as:

\[ \delta P_t^i C_t^i = W_t^i (1 - N_t^i) \]  
(24)

\[ P_t^i C_t^i + E_t \mathcal{F}_{t,t+1}^i \Omega_t^i = P_t^i \mathcal{O}_t^i + \Omega_{t+1}^i \]  
(25)

\[ P_t^i C_t^i = \zeta (\Omega_{t-1}^i + \mathcal{H}_{t-1}^i). \]  
(26)

Solving the system (25)–(26) we derive the dynamic path of aggregate consumption:

\[ P_t^i C_t^i = \sigma E_t \mathcal{F}_{t,t+1}^i \Omega_t^i + \frac{1}{\beta} E_t \mathcal{F}_{t,t+1}^i P_{t+1}^i C_{t+1}^i \]  
(27)

where the first term (in which \( \sigma \equiv \gamma[1 - \beta(1 - \gamma)]/[\beta(1 - \gamma)] \)) captures the financial wealth effect. Notice that the wealth effect fades out as the probability of exiting the market (\( \gamma \)) goes to zero.

2.1.2 Intratemporal Allocation.

At time \( t \), for each household living in country \( i \) and belonging to cohort \( j \), personal consumption is given by the following composite bundle:

\[ C_t^i(j) = \left[ n^\theta C_{H,t}^i(j)^{\frac{\theta - 1}{\theta}} + (1 - n)^{\frac{\theta - 1}{\theta}} C_{F,t}^i(j)^{\frac{\theta - 1}{\theta}} \right]^{\frac{1}{\theta - 1}}, \]  
(28)

in which \( \theta > 0 \) is the elasticity of substitution between Home and Foreign goods, and \( C_{H,t}^i(j) \) and \( C_{F,t}^i(j) \) result from Dixit-Stiglitz-aggregation of the consumption goods produced in the two countries:

\[ C_{H,t}^i(j) = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n C_t^i(h,j)^{\frac{1}{\epsilon}} dh \right]^{\frac{1}{1 - \epsilon}}, \]

\[ C_{F,t}^i(j) = \left[ \left( \frac{1}{1 - n} \right)^{\frac{1}{\epsilon}} \int_n^1 C_t^i(f,j)^{\frac{1}{\epsilon}} df \right]^{\frac{1}{1 - \epsilon}} \]  
(29)

where \( \epsilon > 1 \) is the elasticity of substitution between the differentiated goods in the intervals \([0, n] \) and \((n, 1]\). We assume such elasticity, reflecting the degree of market power, to be the same across countries.

Total expenditure minimization yields the price indexes for goods produced in countries \( H \) and \( F \) and sold in country \( i = H, F \):

\[ P_{H,t}^i = \left[ \frac{1}{n} \int_0^n P_t^i(h)^{1-\epsilon} dh \right]^{\frac{1}{1 - \epsilon}} \quad P_{F,t}^i = \left[ \frac{1}{1 - n} \int_n^1 P_t^i(f)^{1-\epsilon} df \right]^{\frac{1}{1 - \epsilon}} \]  
(30)

---

8 The aggregate per-capita levels across cohorts for each generation-specific variable \( X(j) \) are computed as the weighted average \( X_t \equiv \sum_{j=-\infty}^\infty \gamma(1 - \gamma)^{t-j} X_t(j) \), for both countries.

9 We assume that there is no home bias in consumption. Equation (28) is therefore the same for both countries; as a consequence, the Law of One Price in this setting implies PPP at each point in time, as discussed below.
the consumer-price index (CPI) for country \(i\)

\[
P_t^i = \left[ n(P_{H,t}^i)^{1-\theta} + (1 - n)(P_{F,t}^i)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{31}
\]
and the total demand \(C_t^d(i', j) \equiv nC_t^H(i', j) + (1 - n)C_t^F(i', j)\) for goods produced at home and abroad for generation \(j\)

\[
C_t^d(h, j) = \left( \frac{P_t^h(h)}{P_{H,t}^i} \right)^{-\epsilon} \left( \frac{P_{H,t}^i}{P_t^i} \right)^{-\theta} C_t^W(j) \quad \text{and} \quad C_t^d(f, j) = \left( \frac{P_t^f(f)}{P_{F,t}^i} \right)^{-\epsilon} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\theta} C_t^W(j), \tag{32}
\]
in which \(C_t^W(j) \equiv nC_t^H(j) + (1 - n)C_t^F(j)\) denotes the total world demand for consumption of generation \(j\).

We define the Terms of Trade (ToT) as the relative price of foreign goods in terms of home goods \((S_t \equiv P_{F,t}^i/P_{H,t}^i)\),\(^{10}\) and assume that the Law of One Price (LOP) holds at the brand level: \(P_t^H(i') = \mathcal{E}_t P_t^F(i')\), for all \(i' = h, f\). Moreover, since preferences are identical and there is no home-bias in consumption, the LOP at the brand level implies Purchasing Power Parity (PPP) at each point in time:

\[
P_t^H = \mathcal{E}_t P_t^F, \quad P_t^H = \mathcal{E}_t P_t^F. \tag{33}
\]

Aggregating across cohorts conditions (32) yields:

\[
C_t^d(h) = \left( \frac{P_t(h)}{P_{H,t}^i} \right)^{-\epsilon} \left( \frac{P_{H,t}^i}{P_t^i} \right)^{-\theta} C_t^W(h) \quad \text{and} \quad C_t^d(f) = \left( \frac{P_t(f)}{P_{F,t}^i} \right)^{-\epsilon} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\theta} C_t^W(f), \tag{34}
\]
where \(C_t^W\) denotes world per-capita consumption, and we dropped the country superscript in the relative prices, given PPP.

We assume that the government of each country consumes an exogenously given amount of domestic goods:

\[
G_t(h) = \left( \frac{P_t(h)}{P_{H,t}^i} \right)^{-\epsilon} G_t^H(h) \quad \text{and} \quad G_t(f) = \left( \frac{P_t(f)}{P_{F,t}^i} \right)^{-\epsilon} G_t^F(h). \tag{35}
\]

The aggregate per-capita demand for each country is computed by means of the familiar Dixit-Stiglitz aggregators \(Y_t^H \equiv \left[ n^{-1} \int_0^n Y_t(h) \frac{dh}{1-\theta} \right]^{\frac{1}{1-\theta}}\) and \(Y_t^F \equiv \left[ (1 - n)^{-1} \int_1^n Y_t(f) \frac{df}{1-\theta} \right]^{\frac{1}{1-\theta}}\), in each country \(i\) the following brand-specific and aggregate demands can be obtained:

\[
Y_t^d(i') = C_t^d(i') + G_t(i') = \left( \frac{P_t(i')}{P_{F,t}^i} \right)^{-\epsilon} Y_t^i + \left( \frac{P_t(i')}{P_{F,t}^i} \right)^{-\theta} C_t^W + G_t^i. \tag{36}
\]

\(^{10}\)Notice that, given the definition of terms of trade, the CPI can be also expressed as

\[
P_t^i = P_{H,t}^i \left[ n + (1 - n)S_t^{-1-\theta} \right]^{\frac{1}{1-\theta}} = P_{F,t}^i \left[ nS_t^{\theta-1} + (1 - n) \right]^{\frac{1}{\theta-1}},
\]
for \(i = H, F\).
2.2 The Supply-Side.

Each firm, in each country, has access to a stochastic linear technology $Y_t(i') = A^i_t N_t(i')$, with $i' = h, f$ and $i = H, F$, whose country-specific productivity shock is $A^i_t$. Firms choose labor demand in a competitive labor market by minimizing their total real costs subject to the technological constraint. In equilibrium, for each firm in country $i$, the real marginal cost will hence be

$$MC_{i,t} = \frac{W^i_t}{P^i_{t,t} A^i_t}. \quad (37)$$

Using the brand-specific demand functions (36) and aggregating across domestic brands, we get the aggregate production function for country $H$:

$$Y^H_t \Xi^H_t = A^H_t N^H_t,$$

in which $\Xi^H_t \equiv n^{-1} \int_0^n \left( P^H_t(h)/P^H_{H,t} \right)^{-\epsilon} dh$ captures (second-order) relative price dispersion among domestic firms and $N^H_t \equiv n^{-1} \int_0^n N(h)dh$ is the aggregate per-capita amount of hours worked in country $H$.

Equilibrium in the labor market and the definition of the terms of trade imply that real marginal costs equal

$$MC_{H,t} = \frac{\delta C^H_t}{A^H_t - Y^H_t \Xi^H_t} \left[ n + (1 - n)S^{1-\theta}_t \right]^{\frac{1}{1-\theta}} \quad (38)$$

$$MC_{F,t} = \frac{\delta C^F_t}{A^F_t - Y^F_t \Xi^F_t} \left[ nS^{\theta-1}_t + (1 - n) \right]^{\frac{1}{1-\theta}}. \quad (39)$$

We follow most of the literature in the field and assume that firms set prices according to Calvo’s (1983) staggering mechanism, with $1 - \theta^i$ being the probability for each firm in country $i$ to optimally adjust its price. In equilibrium, this assumption implies a set of familiar New Keynesian Phillips Curves.

2.3 Long-run Equilibrium.

In Appendix A we list the aggregate per capita equilibrium conditions at time $t$. In this section we derive the relations characterizing a zero-inflation steady state.

First of all, the Uncovered Interest Parity implies $r^H = r^F = r$.

From equations (24), (37) and the aggregate production function we get

$$C^H = \frac{h(S)(A^H - Y^H)}{\delta(1 + \mu)} \quad (40)$$

$$C^F = \frac{Sh(S)(A^F - Y^F)}{\delta(1 + \mu)} \quad (41)$$

$$SC^H = \frac{C^F}{A^H - Y^H} = \frac{C^F}{A^F - Y^F}, \quad (42)$$
and the world level of output:

$$Y^W = \frac{A^W + \delta(1 + \mu)G^W}{1 + \delta(1 + \mu)}. \quad (43)$$

Define aggregate nominal dividends in each country as $P^i_tD^i_t = P^i_tY^i_t - W^i_tN^i_t$. Using the definition of non-tradable wealth $\varpi^i$ and equations (20), (25), and (27) we obtain:

$$\varpi^i = P^i_t\left(\frac{Y^i_t - G^i_t}{P^i_t}\right) - rB^i$$  

$$\Omega^i = C^i + \Omega^i - P^i_t\left(\frac{Y^i_t - G^i_t}{P^i_t}\right) + rB^i$$  

$$C^i\beta(1 + r) = \beta\sigma\Omega^i + C^i$$  

$$rB^i = -Z^i = T^i - P^i_tG^i \quad (47)$$

The definition of tradable financial wealth implies $\Omega^i = (1 + r)[B^i_H + B^i_F]$, while the world amount of outstanding debt is denoted by $B \equiv nB_H + (1 - n)B_F$. Hence, using equation (47), aggregation across countries yields:

$$\Omega^W = (1 + r)B \quad (48)$$

$$rB = T^W - G^W = -Z^W. \quad (49)$$

Finally, we can aggregate (46) across countries, and use (48) to get:

$$\beta(1 + r) = \frac{1}{1 - \sigma B/C^W}, \quad (50)$$

which determines, together with equation (49), the world interest rate $r$ and the world amount of outstanding debt $B$, given the world primary deficit $Z^W$.

Therefore, we can write

$$\beta(1 + r) = 1 + \psi, \quad (51)$$

in which

$$\psi \equiv \frac{\sigma B/C^W}{1 - \sigma B/C^W}. \quad (52)$$

Considering for analytical simplicity a symmetric steady state with zero-primary deficit in both countries ($Z^i = 0$ and therefore $B^i = B = 0$), we obtain that $\psi = 0$ and the world interest rate is simply:

$$\beta(1 + r) = 1. \quad (53)$$

Moreover, using (53) into (46) allows to pin down also the steady state aggregate financial wealth for each country $i$, which under the considered assumptions is zero:

$$\Omega^i = (1 + r)(B^i_H + B^i_F) = 0. \quad (54)$$
The above also implies zero-holdings of net foreign assets, defined as the domestic claims on foreign assets net of the foreign claims on domestic assets (in per capita terms):

\[ NF^H = B^H - \frac{1 - n}{n} B^F = 0 \quad NF^F = B^F - \frac{n}{1 - n} B^H = 0. \quad (55) \]

Equation (54), combined with equations (45), allows to derive the steady state level of consumption for the two countries:

\[ C^H = h(S) (Y^H - G^H) \quad (56) \]
\[ C^F = S h(S) (Y^F - G^F). \quad (57) \]

Using equations (40) and (56), ((41) and (57) for country \( F \)), we can pin down the steady state level of output:

\[ Y^i = \frac{A^i + \delta (1 + \mu) G^i}{1 + \delta (1 + \mu)}. \quad (58) \]

Given equations (42), (56), (57), the aggregate demands (36) and the above equation (58), we can also determine the steady state level for the terms of trade:

\[ S = \left( \frac{A^H - G^H}{A^F - G^F} \right)^{1/\theta}. \quad (59) \]

Notice that in a symmetric steady state \((A^H = A^F = A) \text{ and } (G^H = G^F = G)\) the terms of trade is driven to \(S=1\), and symmetry applies also to all other real variables.\(^{11}\)

As a consequence, we also have

\[ \varphi \equiv \frac{N}{1 - N} = \frac{1 + \delta (1 + \mu) G}{\delta (1 + \mu)(1 - \frac{G}{A})}. \quad (60) \]

3 The Linear Model and Macroeconomic Policies.

Let \( x_t \equiv \log X_t - \log X \) denote the log-deviation of variable \( X \) from its steady state, except \( g_t^i \equiv g (G_t^i/G), \tau_t^i \equiv \frac{T^i}{T} \log(T_t^i/T) \), and \( z_t^i \), \( \omega_t^i \) and \( b_{i,t} \), which, given the assumption of

\(^{11}\) Hence:

\[ Y = Y^W = Y^H = Y^F = \frac{A + \delta (1 + \mu) G}{1 + \delta (1 + \mu)}. \]
\[ N = N^W = N^H = N^F = \frac{1 + \delta (1 + \mu) G}{1 + \delta (1 + \mu)} \]
\[ C = C^W = C^H = C^F = \frac{A - G}{1 + \delta (1 + \mu)} \]
\[ r = r^W = r^H = r^F = \frac{1 - \beta}{\beta} \approx - \log \beta \equiv \varrho \]
\[ D = D^W = D^H = D^F = Y \frac{\mu}{1 + \mu} \]
\[ \Omega = \Omega^W = \Omega^H = \Omega^F = 0 \]
\[ S = h(S) = f(S) = 1 \]
\[ T = T^H = T^F = G. \]
zero-primary deficit in steady state, we define as $z_i^t ≡ Z_i^t/C$, $\omega_i^t ≡ \Omega_i^t/C$ and $b_{i,t} ≡ B_{i,t}/C$. Moreover, let $x^W ≡ nx^H + (1 - n)x^F$ denote world aggregates and $x^R ≡ x^H - x^F$ denote $H$ relative aggregates. We also set $s_c ≡ Y/C$.

Log-linearization of equation (24) yields
\[ c_i^t + \varphi n_i^t = w_i^t - p_i^t \]

The linearized version of the UIP is
\[ E_t \Delta e_{t+1} = r_H^t - r_F^t, \]
which, coupled with the LOP and the symmetric preferences over consumption indexes, implies
\[ r_H^t - E_t \pi_H^{t+1} = r_F^t - E_t \pi_F^{t+1}, \]
in which $\pi_i^t ≡ \log(P_i^t/P_i^{t-1})$ is the CPI-based inflation rate for country $i$.

From equations (25), (20), (21) and the definition of aggregate dividends we derive the linear state equation for aggregate financial wealth in country $i$:
\[ \beta \omega_i^t = \omega_i^{t-1} + s_c(y_i^t - g_i^t) - c_i^t + (p_{i,t}^t - p_i^t) + b_{i,t} - \frac{1}{\beta} b_{i,t-1}. \]

Let $b_{k,t} ≡ (B_{k,t} - B_{k})/C$, for $i, k = H, F$. Linearizing the definition (13) implies:
\[ \omega_i^t = \frac{1}{\beta} b_{H,t}^t + \frac{B_H^H}{\beta C}(r_H^t - \varrho) + \frac{1}{\beta} b_{F,t}^t + \frac{B_F^H}{\beta C}(r_F^t - \varrho) + E_t \Delta e_{t+1}, \]
in which $\varrho ≡ -\log \beta$ denotes the steady state real interest rate. Using the UIP in the last term finally implies:
\[ \omega_i^t = \frac{1}{\beta} (b_{H,t}^H + b_{F,t}^F) + \frac{B_H^H + B_F^H}{\beta C}(r_H^t - \varrho) = \frac{1}{\beta} (b_{H,t}^H + b_{F,t}^F), \]
where the last equality is implied by equation (54): zero-holdings of financial assets in steady state makes the interest rate irrelevant for the dynamics of financial wealth and net foreign assets. For country $F$, analogously we have:
\[ \omega_i^F = \frac{1}{\beta} (b_{H,t}^F + b_{F,t}^F). \]

Therefore equation (64) can be written as:
\[ \alpha_i^t = \frac{1}{\beta} \alpha_i^{t-1} + s_c(y_i^t - g_i^t) - c_i^t + (p_{i,t}^t - p_i^t). \]
where $\alpha_i^t \equiv \beta \omega_i^t - b_{i,t} = b_{H,t}^i + b_{F,t}^i - b_{i,t}$ denote the holdings of net foreign assets of residents in country $i$.\footnote{To see this, consider the linear version of equations (22) and (23):}

A second implication is that it must be $n\alpha^H_t + (1-n)\alpha^F_t = 0$, for all $t$. Given the previous implication, and since $p_{H,t}^H - p_{H,t}^F = -(1-n)s_t$ and $p_{F,t}^F - p_{F,t}^H = ns_t$, the law of motion for net foreign assets can be expressed in terms of country $H$’s position as:

$$\alpha^H_t = \frac{1}{\beta} \alpha^H_{t-1} + (1-n)s_t(y_t^R - g_t^R) - (1-n)c_t^R - (1-n)s_t.$$  

(69)

Linearization of the aggregate demands (36) yields:

$$y_t^H = \frac{\theta(1-n)}{sc} s_t + \frac{1}{sc} c_t^W + g_t^H$$  

(70)

$$y_t^F = -\frac{\theta n}{sc} s_t + \frac{1}{sc} c_t^W + g_t^F,$$  

(71)

from which it follows:

$$y_t^W = \frac{1}{sc} c_t^W + g_t^W$$  

(72)

$$y_t^R = \frac{\theta}{sc} s_t + g_t^R.$$  

(73)

Using the last relation, we can reduce the law of motion of net foreign assets to a function of consumption differential and the terms of trade:

$$\alpha^H_t = \frac{1}{\beta} \alpha^H_{t-1} + (\theta - 1)(1-n)s_t - (1-n)c_t^R.$$  

(74)

Let $\pi_{i,t} \equiv \log(P_{i,t}/P_{i,t-1})$ denote the PPI-based inflation rate for country $i$. The following relations hold:

$$\pi_t^H = \Delta e_t + \pi_t^F$$  

(75)

$$\pi_t^H = \pi_{H,t} + (1-n)\Delta s_t$$  

(76)

$$\pi_t^F = \pi_{F,t} - n\Delta s_t.$$

(77)
The above equations, moreover, imply the following law of motion for the terms of trade:

\[ s_t = s_{t-1} + \Delta e_t + \pi_{F,t} - \pi_{H,t}. \]  

(78)

Public debt in country \( i \) evolves according to the following linearized law of motion:

\[ b_{i,t} = \frac{1}{\beta} b_{i,t-1} + z^i_t, \]  

(79)

where \( z^i_t \equiv Z^i_t/C \) denote the real primary deficits:

\[ z_{H,t} = sc(g^H_t - \tau^H_t) - (sc - 1)(1 - n)s_t \]  

(80)

\[ z_{F,t} = sc(g^F_t - \tau^F_t) + (sc - 1)ns_t. \]  

(81)

The state equations for domestic, world and relative consumption read:

\[ c^i_t = E_t c^i_{t+1} - (r^i_t - E_t \pi^i_{t+1} - \varphi) + \sigma \beta \omega^i_t \]  

(82)

\[ c^W_t = E_t c^W_{t+1} - (r^W_t - E_t \pi^W_{t+1} - \varphi) + \sigma b^W_t \]  

(83)

\[ c^R_t = E_t c^R_{t+1} + \sigma b^R_t + \sigma \frac{1}{1 - n} \varphi g^H_t \]  

(84)

in which relative public debt evolves according to \( b^R_t = \frac{1}{\beta} b^R_{t-1} + z^R_t \).

On the supply side, finally, Calvo price-setting implies two NKPC of the usual kind:

\[ \pi^i_{t} = \beta E_t \pi^i_{t+1} + \lambda^i m^i_{c,t}, \]  

(85)

in which the real marginal costs, expressed in terms of aggregate and relative variables, follow:

\[ m^H_{c,t} = \frac{s_c + \varphi}{s_c} c^W_t + (1 - n) c^R_t + (1 - n) \frac{s_c + \varphi \theta}{s_c} s_t + \varphi g^H_t - (1 + \varphi) a^H_t \]  

(86)

\[ m^F_{c,t} = \frac{s_c + \varphi}{s_c} c^W_t - n c^R_t - n \frac{s_c + \varphi \theta}{s_c} s_t + \varphi g^F_t - (1 + \varphi) a^F_t. \]  

(87)

3.1 The Policy Makers

In each country two policy makers act: a Central Bank and a fiscal authority.

Central Banks are in charge of monetary policy. In this paper we do not explicitly address issues related to welfare. Hence, the monetary policy regime that we use as benchmark in both countries is the “Wicksellian” Monetary Policy, targeting the Natural Rate, \( r^i_t \), and leading to price stability. A credible threat to deviate from \( r^i_t \) rules out indeterminacy and implements the flexible-price allocation:  

\[ r^i_t = \frac{1}{\beta} r^i_{t-1} + \alpha^i_t \pi^i_{t}. \]  

(88)

\[ \frac{1}{\beta} \]  

We restrict our attention to active monetary policy, satisfying the Taylor principle (\( \phi^i_\pi > 1 \), for all \( i = H, F \)).
Governments are in charge of fiscal policy. We assume they use real taxes as an instrument to meet their policy targets, given an exogenous stationary process for public spending: 
\[ g_i^t = \rho_i^g g_i^{t-1} + u_{i,t}. \]
Our benchmark specification for fiscal policy follows some recent empirical literature (e.g. Gali and Perotti, 2003) and has the target primary deficit vary countercyclically, according to a feedback rule of the following kind:

\[ z_i^t = \rho_i^z z_i^{t-1} - (1 - \rho_i^z) [\mu_i^b b_{i,t-1} + \mu_i^x x_i^t] + u_{i,t}, \tag{89} \]

where all response coefficients are positive and \( \rho_i^z \in [0, 1). \)

As argued in the introduction, the reason for choosing this kind of specification for fiscal policy lies in its flexibility to nest several fiscal regimes which may characterize modern industrialized economies and in its widespread use in the recent empirical literature. In this perspective, an additional goal of our analysis is to evaluate the theoretical properties of such a deficit rule in a fully fledged DSGE model with endogenous interaction between fiscal and external balances.

### 3.2 The Flexible-Price Equilibrium

Let \( \bar{m}_t \) denote the level of generic variable \( m_t \) in the flexible-price equilibrium at time \( t \). Given stochastic processes for \( a_i^t \) and \( g_i^t \), the complete linear model under flexible prices and counter-cyclical deficit rules becomes:

\[ \Delta \bar{s}_t = \Delta \bar{s}_t \tag{90} \]
\[ \bar{s}_t^H = \frac{1}{\beta} \bar{s}_t^{H-1} + (\theta - 1)(1-n)\bar{s}_t - (1-n)\bar{s}_t^R \tag{91} \]
\[ \bar{s}_t^W = E_t \bar{s}_{t+1}^W - \bar{s}_t^W + \sigma \bar{s}_t^W \tag{92} \]
\[ \bar{s}_t^R = E_t \bar{s}_{t+1}^R + \sigma \bar{s}_t^R + \frac{\sigma}{1-n}\bar{s}_t^H \tag{93} \]
\[ \beta \bar{s}_t^H - \beta \bar{s}_t^H = \bar{s}_{t-1}^H \tag{95} \]
\[ \beta \bar{s}_t^F - \beta \bar{s}_t^F = \bar{s}_{t-1}^F \tag{96} \]

\[ 0 = \frac{s_c + \varphi}{s_c} \bar{c}_t^W + (1-n)\bar{s}_t^R + (1-n)\frac{s_c + \varphi \theta}{s_c} \bar{s}_t^R + \varphi g_t^H - (1 + \varphi) a_t^H \tag{97} \]
\[ 0 = \frac{s_c + \varphi}{s_c} \bar{c}_t^W - n \bar{s}_t^R - n \frac{s_c + \varphi \theta}{s_c} \bar{s}_t^R + \varphi g_t^F - (1 + \varphi) a_t^F \tag{98} \]

\[ \text{Also with respect to this policy rule, restrictions on the response coefficients must be satisfied in order for the equilibrium to be determinate. In particular, given the assumption of active monetary policy, to rule out the unstable root implicit in the law of motion of public debt (79) the response coefficient to the stock of outstanding debt must be larger than the steady state net interest rate, in each country: } \mu_i^b > \varrho, \text{ for all } i = H, F. \]
\[ z_t^H = \rho_z z_{t-1}^H - (1 - \rho_z^H) \mu_b b_{t-1}^H + u_{z,t}^H \quad (99) \]
\[ z_t^F = \rho_z z_{t-1}^F - (1 - \rho_z^F) \mu_b b_{t-1}^F + u_{z,t}^F. \quad (100) \]

In the flexible price equilibrium, marginal costs are zero in both countries, and so are domestic inflation rates. The condition \( mc_{t,H} = mc_{t,F} = 0 \) implies (aggregating across countries):
\[ \bar{\tau}_t = \frac{s_c}{s_c + \varphi} \left( (1 + \varphi) a_t^H - \varphi g_t^H \right). \quad (101) \]

Moreover, the same condition also implies:
\[ \bar{\tau}_t = \frac{s_c}{s_c + \varphi \theta} (1 + \varphi) a_t^H - \varphi g_t^H. \quad (102) \]

As to the remaining conditions, the relevant system to be studied is the following 6-by-6:
\[ E_t \bar{c}_{t+1}^R + \sigma (\bar{b}_t^H - \bar{b}_t^F) + \frac{\sigma}{1 - n} \bar{\pi}_t^H = \bar{c}_{t}^R \quad (103) \]
\[ \beta \bar{\pi}_t^H = \bar{\tau}_{t-1}^H - (1 - n) \beta \theta \frac{s_c + \varphi \theta}{s_c + \varphi} \bar{\pi}^R_t \]
\[ + \beta (\theta - 1)(1 - n) \frac{s_c}{s_c + \varphi \theta} [1 + \varphi] a_t^H - \varphi g_t^R \]
\[ \beta \bar{b}_t^H - \beta \bar{z}_t^H = \bar{b}_{t-1}^H \quad (104) \]
\[ \beta \bar{b}_t^F - \beta \bar{z}_t^F = \bar{b}_{t-1}^F \quad (105) \]
\[ \bar{z}_t^H = \rho_z^H z_{t-1}^H - (1 - \rho_z^H) \phi_b b_{t-1}^H + u_{z,t}^H \quad (106) \]
\[ \bar{z}_t^F = \rho_z^F z_{t-1}^F - (1 - \rho_z^F) \phi_b b_{t-1}^F + u_{z,t}^F. \quad (107) \]

Provided that the conditions on the response coefficients to the stock of outstanding debt in the fiscal rules are satisfied, the system above delivers the Wicksellian (or natural) values for the relative consumption \( \bar{\pi}^R_t \), the net foreign asset holdings \( \bar{\pi}_t^H \), the two deficits \( \bar{z}_t^i \) and the stocks of debt \( \bar{b}_t^i \), for \( i = H, F \).

### 4 Simple Policy Rules and Current Account Dynamics

In this section we simulate our model and study the macroeconomic implications of different combinations of monetary and fiscal policies.

We study six alternative monetary policy specifications. The benchmark regime is the “Wicksellian” Monetary Policy:
\[ r_t^i = \bar{\pi}_t^i + \phi_\pi^i \pi_{i,t}, \quad (109) \]
in which \( \phi_\pi^i > 1 \), for all \( i = H, F \). We then compare the dynamic and cyclic properties of several simple rules to the ones featured by this benchmark.
The first two simple rules considered are standard interest rate rules of the kind introduced by Taylor (1993). One has the nominal interest rate respond to deviations of the GDP deflator $\pi_{t,H}$ from the zero target (DITR):

$$r_{t,H} = \varrho + \phi_{H}^{\pi} \pi_{t,H} + u_{r,t}^{H},$$  \hspace{1cm} (110)

in which $u_{r,t}^{H}$ are white noises capturing pure monetary policy shocks. The second (CITR) modifies equation (110) by allowing for a reaction to variations in the CPI inflation rate $\pi_{t}^{H}$:

$$r_{t,H} = \varrho + \phi_{H}^{\pi} \pi_{t}^{H} + u_{r,t}^{H}. \hspace{1cm} (111)$$

In the quantitative analysis below, we parameterize both rules with a reaction coefficient $\phi_{H}^{\pi} = 2$.

A different monetary regime is based on the control of the nominal exchange rate by the Central Bank, possibly in the attempt of correcting external imbalances. In this regime, the exchange rate is devaluated when the stock of outstanding net foreign assets falls below its potential level:

$$e_{t} = -\chi_{H}(\alpha_{t-1} - \pi_{t,H}) + u_{e,t}^{H},$$  \hspace{1cm} (112)

in which $u_{e,t}^{H}$ are white noises capturing again pure monetary policy shocks. With respect to this regime, we identify three stances, depending on the value assigned to the response coefficient: $\chi_{H} = 0$ denotes a pure exchange rate peg, in which the Home Central Bank keeps $e_{t} = 0$ at all times (PEG); we then consider the case in which the Home Central Bank controls the nominal exchange rate to correct external imbalances, in a “passive” way, when $\chi_{H} = 1$ (FLOPAS) and in an “active”, aggressive way, in which the response coefficient is increased to $\chi_{H} = 6$ (FLOACT).

As to fiscal policy, we consider four alternative specifications, focusing only on “passive” (in the sense of Leeper, JME 1991) or implementable (in the sense of Schmitt-Grohe and Uribe, 2006) fiscal rules.

First, we consider the case in which governments have no specific target.\textsuperscript{15} This specification (TX) implies that not only $g_{t}$, but also real taxes follow an exogenous, stationary autoregressive process:

$$\tau_{t}^{H} = \rho_{\tau}^{H} \tau_{t-1}^{H} + (1 - \rho_{\tau}^{H})b_{t-1}^{H} + u_{\tau,t}^{H},$$  \hspace{1cm} (113)

in which a drift adjusting to the stock of outstanding debt insures equilibrium determinacy ($\xi_{b}^{H} = (\phi/s_c)$) and fiscal solvency.

The second specification (BB) considers the case in which the government targets a balanced budget in every period:

$$z_{i}^{1} = 0, \hspace{1cm} (114)$$

\textsuperscript{15}This specification may proxy the way the U.S. administration conducted fiscal policy in the ’80s, as the empirical evidence in Gali and Perotti (2003) suggests. Note, in fact, that the regime TX is equivalent to our benchmark deficit feedback rule (89), when the response coefficient to the output gap is zero.
while in the third (DS) implies full stabilization of the debt-to-GDP ratio:

\[ b_t^i = y_t^i. \]  

(115)

Our benchmark specification considers the case in which governments set their primary deficit following a counter-cyclical feedback rule (89) of the kind:

\[ z_t^i = \rho^i z_{t-1}^i - (1 - \rho^i_z) [\mu^i_b b^i_{t-1} + \mu^i_x x^i_t] + u^i_{z,t}. \]  

(116)

We calibrate the parameters above using the estimates provided by Galì and Perotti (EP, 2003) for the period 1992-2001. We identify the U.S. as the \( H \) country and the group of EMU10 as the \( F \) country: \( \rho^H_z = 0.25, \rho^F_z = 0.42, \mu^H_x = 1.43, \mu^F_x = 0.47, \mu^F_b = 0.07. \) As to the Home response to the existing stock of debt, \( \mu^H_b \), we consider two alternative degree of fiscal discipline: a “low” and a “high” degree. “Low” fiscal discipline implies a policy rule in which the response coefficient to the stock of outstanding debt is set at a level slightly higher than the one necessary to grant solvency and determinacy: \( \mu^H_b = 1.5\varrho. \) This calibration implies a response coefficient to existing debt of about .015, and is consistent with the estimates of Galì and Perotti (EP, 2003) for many OECD countries\(^1\) and with the estimate provided by Favero and Monacelli (2005) for the U.S. for the 90’s. As “High” degree of fiscal discipline we consider \( \mu^H_b = 15\varrho, \) which implies a response of about .15, and is consistent with the evidence of countries with a virtuous debt dynamics like Australia.

In the next subsections, we compare the dynamic and cyclical properties of alternative combinations of monetary and fiscal policies for the Home economy. We will constantly assume that the foreign authorities follow our defined benchmark regimes (i.e. the “Wicksellian” Monetary Policy and the counter-cyclical deficit feedback rule (116), calibrated as discussed above).

### 4.1 Calibration

We parameterize the model on a quarterly frequency, following previous studies and convention. Specifically, the steady-state net quarterly interest rate \( \varrho \) was set at 0.01, implying a long-run real annualized interest rate of 4%.\(^2\) The probability of exiting the markets \( \gamma \) was set at 0.05. In order to meet the steady-state restrictions, the intertemporal discount factor \( \beta \) was set at 0.99. The elasticity of substitution among intermediate goods \( \epsilon \) was set at 11, \(^{16}\)

\(^{16}\)The empirical analysis in Galì and Perotti (2003) also provides estimates for the 1980-1991 period, which show that in the ’80s the response coefficients \((\mu^H_b, \mu^H_x)\) for the U.S. where both insignificant. This case broadly corresponds to the fiscal regime that we labeled TX.

\(^{17}\)The estimate that Galì and Perotti (EP, 2003) report for the U.S. is actually not significantly different from zero, over the whole 1980-2001 sample.

\(^{18}\)Since we concentrate on a symmetric steady state the values reported in the text are meant to refer to both countries as well as to the world economy.
implying a steady-state net mark-up rate of 10%, the elasticity of substitution between Home and Foreign goods was set equal to 1.5, while the probability for firms of having to keep their price fixed for the current quarter was set at 0.75 for both countries, implying that prices are revised on average once a year. As to the steady-state Frisch elasticity of labor supply, $1/\varphi$, there is wide controversy about the value that should be assigned to this parameter. The empirical microeconomic literature suggests values for $\varphi$ ranging from .1 to .5 (see Card, 1994, for a survey), while business cycle literature mostly uses values greater than 1 (see e.g. Cooley and Prescott, 1995). We choose a baseline value of $\varphi \equiv \frac{N}{1-N} = 1$; given the steady state restrictions derived in the appendix and the implied value of $\mu$, this requires a value of $\delta$ equal to 1.125. Finally, we parameterize the dimension of the Home country $n$ to 0.6, roughly consistent with the ratio of U.S. to Euro-10 GDP.

4.2 The Dynamic Response to Structural Shocks

4.2.1 Productivity Shocks

We start by studying the macroeconomic effects of a positive domestic productivity shock, under alternative fiscal and monetary regimes (figures 1–4). More precisely, figures 1–3 focus on the three exogenous fiscal regimes (TX, BB, and DS) and compare the Wicksellian Monetary Policy with the Domestic Inflation-based Taylor rule. In Figure 4, which considers alternative endogenous deficit feedback rules, we add a third monetary regime, based on aggressive managed floating. We do not report in these figures the other three monetary regimes explored (CITR, PEG, FLOPAS) because they prove inferior in replicating the Wicksellian Policy, with respect to the DITR. We explore this issue further in the next session, when investigating the cyclical properties of the alternative regimes.

The effect of a local, positive, productivity shock on domestic inflation, the output gap and the exchange rate is qualitatively similar across all fiscal regimes. Under flexible prices, such effect is an improvement in the terms of trade, through the relative reduction in domestic interest rates and the following depreciation of the exchange rate. On impact, then, the economy experiences a current account surplus, through higher competitiveness, because residents in the Home country accumulate foreign assets to smooth the effects of the shock on consumption. Under sticky prices, these dynamic effects are closely mirrored if the Central Bank adopts a simple interest rate rule. However, in the case of aggressive managed floating domestic interest rates rise on impact to prevent the depreciation of the exchange rate, thus causing a deeper fall in the output gap (see figure 4). When a surplus in the natural current account starts accumulating, the Central Bank accommodates such dynamics by lowering interest rates so as to depreciate the exchange rate and support higher levels of output and inflation.

When it comes to the reaction of the other variables of interest, the actual fiscal regime
adopted becomes relevant.

Figure 1 shows that, when the government has no target in terms of primary deficit (TX), the improvement in the terms of trade stimulates a primary fiscal surplus, through the reduction of home relative prices (at which public purchases are made). Taxes move slightly, to ensure a non-explosive path to public debt. On impact, net foreign assets are accumulated given higher relative productivity. In the long run, the dynamics of public debt becomes the dominant factor for the evolution of national savings, and governs the speed of convergence of NFA.

Figure 2 shows the case of a balanced budget target for the government (BB). In this case, the effect due to competitiveness is the only one at play, given that no debt is issued, and the dynamics of NFA are mean reverting, like the one of the terms of trade (not showed). Analogously, when the government seeks to stabilize the debt-to-GDP ratio (figure 3), the net foreign assets revert to their steady state level, driven by the terms of trade, while primary deficit and public debt jump on impact to match the increase in real output induced by the productivity shock.

Turning to the endogenous fiscal regimes, figure 4 compares the dynamic outcomes of following “low” versus “high” fiscal discipline. Under the “Wicksellian” regime, a positive productivity shock produces a surplus in the current account, which is gradually absorbed over time. Given that the output gap is unaffected, both the primary deficit and the stock of public debt do not move (only real taxes slightly fall, in order to sterilize the initial improvement in the terms of trade).

If monetary policy is conducting according to simple policy rules inflation and output gap decrease on impact, to revert to their equilibrium values over time, and the improvement in the terms of trade is weaker. The fiscal rule requires real taxes to fall, thereby increasing the stock of public debt.

Under DITR, “Low” fiscal discipline results in public debt accumulation (DITR-Low). Both public and private savings fall, the latter through expansionary wealth effects on consumption. Eventually, the fall in national savings overrules the initial improvement in the terms of trade, and the country moves to a persistent external deficit. Notice that, in presence of “Low” fiscal discipline, monetary policy could prevent the current account deficit from developing, by aggressively managing the exchange rate (FLOACT-Low). However such a policy would require a strong and persistent depreciation of the exchange rate, with high costs in terms of both inflation and output.

On the other hand, a “High” degree of fiscal discipline allows the government to drive both NFA and the stock of outstanding debt back to their equilibrium values. This result holds regardless of the monetary policy rule adopted.\(^\text{19}\) This is due to the fact that under

\(^{19}\)Not showed in the figure. An appendix with all the impulse response functions is available upon request.
“High” fiscal discipline real taxes would adjust to produce a moderate primary surplus before the growth of public debt starts overruling the terms-of-trade effect. The degree of fiscal discipline therefore plays a crucial role in the dynamics of net foreign assets, especially in the medium run.

4.2.2 Fiscal and Monetary Policy Shocks

We now move to the dynamic effects of a tax cut on the home economy, when the government does not set any specific target in terms of its budget deficit (figure 5). We take this scenario to approximate, in the context of our stylized model, the situation in the U.S. at the beginning of the 1980s, when the Reagan administration approved the largest tax cut in American history (while apparently not following any deficit feedback rule) and the twin deficits first appeared.

In our model, given the level of government expenditure, the tax cut produces a fiscal deficit (the first twin) and the issuance of new debt to finance it. On impact, the world and relative stocks of outstanding debt increase, as well as world and relative consumption, through wealth effects. The increase in relative consumption then induces upward pressures on relative marginal costs, which require an increase in relative nominal interest rates (and the following appreciation in the nominal exchange rate and deterioration of the terms of trade). Under “Wicksellian” Monetary Policy the increase in the nominal interest rate would be sufficient to sterilize the effects on output and inflation, and the appreciation would be maximum; under a simple Taylor rule, instead, the sterilization would only be partial, inflation would rise and the terms of trade would deteriorate more, driving a slight reduction in the output gap.

These events trigger the gradual accumulation of a current account deficit (the second twin). Overtime, the near-unit root in debt dynamics implied by “fiscal indiscipline” generates a persistent accumulation in the stock of public debt, which then translates persistently into: negative external imbalances, depreciation of the exchange rate and eventually a negative relative consumption (when the negative effect of the current account deficit overrules the positive effect of accumulating public debt, according to equation (84)).

Although working through a different mechanism, therefore, we obtain the same qualitative results in terms of dynamics as Ganelli (JIE, 2005). When moving to analyze an expansion in public consumption, however, Ganelli (JIE, 2005) shows that the effects on the exchange rate depend on how the expansion is financed: a tax-financed expansion (BB) would imply an on-impact depreciation through a reduction in relative consumption and an increase in domestic prices, as in the Redux model, while the effects of a debt-financed expansion would be ambiguous. He argues that this happens because “all the costs [...] in terms of taxes are sustained by domestic residents [while] the benefits [...] in terms of increased
public demand for privately produced goods, are shared with foreigners”. Indeed, this result follows directly from the assumption that public expenditure is uniformly distributed over the continuum of privately produced goods, both domestic and foreign. An increase in public expenditures can therefore be interpreted as a global demand shock.

In our setting, government spending is only on domestic goods and services, and both direct costs and benefits therefore fall on domestic residents. The demand shock is country-specific and an increase in relative public consumption tends to raise relative marginal costs (equations (86)–(87)), thereby triggering an increase in relative interest rates to offset the inflationary pressures. This results in a short-run appreciation of the nominal exchange rate, which worsens the external position. As a consequence, relative consumption falls (equation (84)). In our economy, therefore, the final short-run effect on relative consumption and net foreign assets are the same as in Ganelli (JIE, 2005) and Obstfeld–Rogoff (JPE, 1995), while the effects on the exchange rate are reversed (figure 6).

It also follows that a debt-financed expansion in public consumption (figure 7) unambiguously induces an appreciation of the nominal exchange rate on impact (unlike in Ganelli, JIE 2005) and a depreciation in the transition.

Figures 8 and 9 compare the dynamic outcomes of following “Low” versus “High” fiscal discipline in the face of different fiscal shocks. We show that (like in the case of productivity shocks) adopting a “Low” degree of fiscal discipline leaves room for perverse dynamics of net foreign assets, unless the Central Bank aggressively manages the exchange rate. In the case of a debt-financed tax-cut, moreover, we show that the degree of fiscal discipline substantially affects also the exchange rate dynamics. By inducing a higher persistence in external imbalances, in fact, a “Low” degree of fiscal discipline also exacerbates the fluctuations in the nominal exchange rate.

Finally, figure 10 shows the effects of a (white-noise) monetary policy shock. In the case of endogenous, counter-cyclical fiscal policy, a pure monetary policy shock affects inflation and the output gap, thereby triggering a reaction by the fiscal authority. Once the fiscal deficit is moved, then, fiscal discipline is bound to rule the medium-run dynamics of net foreign asset and public debt, which are going to revert back to mean, the faster the more disciplined the government. Interestingly, therefore, figure 10 suggests that a fiscal authority with a low degree of fiscal discipline might leave room for growing and persistent external imbalances spurred also by temporary monetary policy shocks.

---

20This assumption follows most of the recent literature and seems roughly consistent with empirical evidence. Di Giorgio–Nisticò (2007a) study analytically the role of home bias in public consumption for the exchange rate dynamics, in an extension of the Redux model.
4.3 Implied Volatilities.

Table 1 reports the standard deviations, in percentage points, of selected variables implied by all combinations of monetary and fiscal policy rules. For each entry (except for the Wicksellian policy) a bold figure indicates the relative performance with respect to a benchmark regime: with respect to domestic inflation and the output gap (driven to zero under Wicksellian policy), the comparison is made with the Domestic Inflation-based Taylor Rule, while for all other variables the bold figure reports the ratio with respect to the corresponding standard deviations implied by the Wicksellian regime. For the entries corresponding to low fiscal discipline (Low), we also report the relative volatility loss implied by a higher response to debt dynamics (ratio Low-to-High). Table 1 suggests the following.

First. Regarding the volatility of inflation and the output gap, the monetary policy interest rate rule reacting to domestic inflation (DITR) proves superior to all the other ones in approximating the Wicksellian regime, regardless of the fiscal regime in place. The third and fourth panels show the – familiar – result that reacting to CPI inflation or pegging the nominal exchange rate generates excessive smoothness in the exchange rate and the terms of trade relative to the benchmark case, and yields amplified fluctuations for both inflation and the output gap. This is why we mainly discussed impulse responses for the DITR and FLOACT cases.

Second. Moving from controlling an interest rate to managing the exchange rate allows to better approximate the volatility of net foreign assets to the one obtained under the Wicksellian policy, regardless of the fiscal regime in place.

Third. Under the TX fiscal regime, both the stocks of net foreign assets and public debt experience huge volatilities, due to the near-unit root in the dynamics of the latter.

Fourth. Fiscal discipline pays off in terms of stability gain for both debt and net foreign assets, without any cost for the volatility of inflation and the output gap. In particular, the volatility of net foreign assets under “High” fiscal discipline is reduced by more than one half.

Fifth. Comparing the cyclical properties of the policy mix consisting of aggressive managed float and poorly disciplined deficit rule (FLOACT-Low) with the ones of the alternative mix “DITR-High” shows that all endogenous variables are more stable if monetary policy deals with inflation and output gap and the fiscal authorities behave rigorously. As a matter of fact, the policy mix “FLOACT-Low” shows some ability in controlling the dynamics of the external balance, but it implies much higher volatility of inflation, the exchange rate and the output gap.

\[\text{\textsuperscript{21}In terms of the volatility of net foreign assets, the excess smoothness of the exchange rate passes through to the external balance under “High” fiscal discipline. Under “Low” fiscal discipline, such volatility is strongly affected by the dynamics of public debt.}\]
Concluding Remarks.

This paper lays out a tractable stochastic two-country “perpetual youth” DNK model, which achieves determinacy of steady state net foreign asset holdings, and allows for endogenous interaction of the external balance with fiscal and monetary policies. Within this framework we analyze the theoretical properties of deficit feedback rules (in line with most of the recent empirical literature) and study a wide range of fiscal and productivity shocks under alternative combinations of monetary and fiscal policy.

We find that “fiscal discipline”, defined as the degree of concern about public debt dynamics on the part of the fiscal authority, plays an important role for net foreign asset dynamics and exchange rate determination. Moving from a fiscal regime in which the budget deficit is set “exogenously” to one in which it “endogenously” responds to the business cycle (as the empirical evidence suggests for the 1990’s) may induce a transmission mechanism that amplifies the distortions in the system, following a structural shock. In such circumstances, persistent deteriorations of the external balance may spur from fiscal expansions as well as from positive productivity shocks, and even after temporary monetary restrictions, unless an appropriate systematic response is granted to public debt dynamics (“High” fiscal discipline). Interestingly, however, the available empirical evidence does not support the view that such an appropriate response has characterized recent fiscal behavior of most industrialized countries, first and foremost the U.S. Any attempt by monetary policy alone to stabilize the external balance could prove somehow effective but would require excessive fluctuations in the exchange rate and imply high costs in terms of inflation and output gap volatility.

Consistently with existing literature on monetary policy, we also find that, with respect to inflation and output gap volatility, interest rate rules reacting to domestic inflation perform better than the alternatives considered, regardless of the fiscal regime in place.
References


A Appendix

A.1 The Complete Model.

At time $t$, therefore, the aggregate per-capita equilibrium conditions read

\[ \delta P^i_t C^i_t = W^i_t (1 - N^i_t) \] (A.1)

\[ P^i_t C^i_t + E_t F^i_{t,t+1} \Omega^{i*}_t = P^i_t \omega^i_t + \Omega^{i*}_{t-1} \] (A.2)

\[ \omega^i_t = \frac{W^i_t}{P^i_t} N^i_t + D^i_t - T^i_t \] (A.3)

\[ P^i_t C^i_t = \sigma E_t F^i_{t,t+1} \Omega^{i*}_t + \frac{1}{\beta} E_t F^i_{t,t+1} P^i_t C^i_{t+1} \] (A.4)

\[ Y^i_t = \left( \frac{P^i_{t,t}}{P^i_t} \right)^{-\theta} C^W_t + G^i_t \] (A.5)

\[ B^*_{t,t} = (1 + r^i_{t-1}) B^*_{t,t-1} + Z^*_{t} \] (A.6)

\[ nB^H_{H,t} = nB^H_{H,t} + (1 - n) B^F_{H,t} \] (A.7)

\[ (1 - n) B^F_{F,t} = nB^H_{H,t} + (1 - n) B^F_{F,t} \] (A.8)

\[ P^i_t Z^i_t = P^i_t G^i_t - P^i_t T^i_t \] (A.9)

\[ MC^i_{t,t} = \frac{W^i_t}{P^i_{t,t} A^i_t} \] (A.10)

\[ \Xi^i_t Y^i_t = A^i_t N^i_t \] (A.11)

\[ P^i_t D^i_t = P^i_{t,t} Y^i_t - W^i_t N^i_t \] (A.12)

\[ \frac{P^H_{H,t}}{P^F_t} \left[ \frac{nS_t^{\theta - 1} + (1 - n) T^F_{t,1} - T^H_{t,1}}{1 - T^H_{t,1}} \right] = h(S_t) \] (A.13)

\[ E_t F^H_{t,t+1} \left[ (1 + r^F_{t,F}) - (1 + r^H_{t,F}) \right] = 0 \] (A.15)

\[ S_t = \frac{P^F_{F,t}}{P^H_{H,t}} \] (A.16)

\[ Y^H_t - G^H_t = S^i_t (Y^F_t - G^F_t) \] (A.17)

\[ C^W_t = nC^H_t + (1 - n) C^F_t \] (A.18)

\[ Y^W_t = nh(S_t) Y^H_t + (1 - n) S_t h(S_t) Y^F_t \] (A.19)

\[ G^W_t = nh(S_t) G^H_t + (1 - n) S_t h(S_t) G^F_t \] (A.20)

\[ Y^W_t = C^W_t + G^W_t. \] (A.21)
After linearization around a zero-inflation, zero-deficit, symmetric steady state, therefore, the equations needed to study the equilibrium (given stochastic processes for $g_t^i$ and $a_t^i$) are the following:

\begin{align}
    s_t &= s_{t-1} + \Delta e_t + \pi_{F,t} - \pi_{H,t} \\
    \alpha_t^H &= \frac{1}{\beta} \alpha_{t-1}^H + (\theta - 1)(1-n)s_t - (1-n)c_t^R \\
    c_t^W &= E_t c_{t+1}^W - (r_t^W - E_t \pi_{W,t+1} - \phi) + \sigma b_t^W \\
    c_t^R &= E_t c_{t+1}^R + \sigma b_t^R + \frac{\sigma}{1-n} \alpha_t^H \\
    b_t^H &= \frac{1}{\beta} b_{t-1}^H + z_t^H \\
    b_t^F &= \frac{1}{\beta} b_{t-1}^F + z_t^F \\
    r_t^R &= r_t^H - r_t^F = E_t \Delta e_{t+1} \\
    \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \lambda^H mc_{H,t} \\
    \pi_{F,t} &= \beta E_t \pi_{F,t+1} + \lambda^F mc_{F,t} \\
    mc_{H,t} &= \frac{s_c + \varphi}{s_c} c_t^W + (1-n)c_t^R + (1-n) \frac{s_c + \varphi}{s_c} s_t + \varphi g_t^H - (1 + \varphi)a_t^H \\
    mc_{F,t} &= \frac{s_c + \varphi}{s_c} c_t^W - nc_t^R - n \frac{s_c + \varphi}{s_c} s_t + \varphi g_t^F - (1 + \varphi)a_t^F \\
    z_{H,t} &= s_c(g_t^H - \tau_t^H) - (s_c - 1)(1-n)s_t \\
    z_{F,t} &= s_c(g_t^F - \tau_t^F) + (s_c - 1)ns_t.
\end{align}

Closing the model with two monetary policy rules, determining either the nominal interest rate or the exchange rate, and two fiscal rules determining either the path of primary deficits or real taxes, and specifying some initial conditions for the position in net foreign asset, public debts and the terms of trade ($\alpha_{-1}^H$, $b_{-1}$, and $s_{-1}$), we get a system of 17 stochastic difference equations, which yields as a solution the equilibrium values of the 17 endogenous variables:

$$\{s_t, \alpha_t^H, b_{H,t}, b_{F,t}, c_t^W, c_t^R, \Delta e_t, \pi_{H,t}, \pi_{F,t}, mc_{H,t}, mc_{F,t}, z_t^H, z_t^F, \tau_t^H, \tau_t^F, r_t^H, r_t^F\}_{t=0}^\infty$$

Considering then $c_t^W = c_t^H - (1-n)c_t^R = c_t^R + nc_t^R$ and the domestic demand schedules (70)–(71), we can finally back out also $c_t^H$, $c_t^F$, $y_t^H$, $y_t^F$, and $y_t^W$. 

31
Figure 1: Dynamic responses in the H-country to a Home productivity shock when the Home Government has no target for the primary deficit.
Figure 2: Dynamic responses in the H-country to a Home productivity shock when the Home Government runs balance budgets in every period.
Figure 3: Dynamic responses in the H-country to a Home productivity shock when the Home Government stabilizes the debt-to-GDP ratios.
Figure 4: Dynamic responses in the H-country to a Home productivity shock when the Home Government follows a counter-cyclical deficit feedback rule (Low and High).
Figure 5: Dynamic responses in the H-country to a Tax-cut when the Home Government has no target for their primary deficit.
Figure 6: Dynamic responses in the H-country to a Balanced-Budget expansion in Home Public Consumption.
Figure 7: Dynamic responses in the H-country to a Debt-Financed expansion in Home Public Consumption and no target for the primary deficit.
Figure 8: Dynamic responses in the H-country to a Home Demand shock when the Home Government follows a counter-cyclical deficit feedback rule (Low and High).
Figure 9: Dynamic responses in the H-country to a Home deficit shock when the Home Government follows a counter-cyclical deficit feedback rule (Low and High).
Figure 10: Dynamic responses in the H-country to a Home monetary shock when the Home Government follows a counter-cyclical deficit feedback rule (Low and High).
<table>
<thead>
<tr>
<th>Monetary Regime</th>
<th>Wicksellian</th>
<th>DITR</th>
<th>CITR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiscal Regime</td>
<td>BB</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic Infl.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI Inflation</td>
<td>0.29</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nom. Int. Rates</td>
<td>0.19</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depreciat. Rate</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terms of Trade</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Govt Deficit</td>
<td>0.00</td>
<td>0.52</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public Debt</td>
<td>0.00</td>
<td>6.65</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Foreign Asset</td>
<td>1.49</td>
<td>2.81</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deficit-to-GDP</td>
<td>0.90</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>0.90</td>
<td>6.71</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NFA-to-GDP</td>
<td>1.60</td>
<td>2.88</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Entries are standard deviations in %. Country F follows the Wicksellian Monetary Rule and the Deficit Feedback Rule.
<table>
<thead>
<tr>
<th>Monetary Regime Fiscal Regime</th>
<th>BB Low</th>
<th>PEG High</th>
<th>DS TX</th>
<th>BB Low</th>
<th>FLOPAS High</th>
<th>DS TX</th>
<th>BB Low</th>
<th>FLOACT High</th>
<th>DS TX</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output Gap</strong></td>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
<td>0.65</td>
<td></td>
<td>0.77</td>
<td>0.78</td>
<td>0.77</td>
</tr>
<tr>
<td>Ratio to DITR</td>
<td>2.23</td>
<td>2.17</td>
<td>2.11</td>
<td>2.23</td>
<td>2.31</td>
<td></td>
<td>2.72</td>
<td>2.69</td>
<td>2.71</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>1.00</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Domestic Infl.</strong></td>
<td>0.20</td>
<td>0.19</td>
<td>0.20</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.17</td>
<td>0.18</td>
<td>0.17</td>
</tr>
<tr>
<td>Ratio to DITR</td>
<td>1.82</td>
<td>1.85</td>
<td>1.82</td>
<td>1.84</td>
<td>0.89</td>
<td>0.98</td>
<td>1.59</td>
<td>1.81</td>
<td>1.58</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>0.98</td>
<td>1.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CPI Inflation</strong></td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.12</td>
<td>0.35</td>
<td>0.37</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>0.39</td>
<td>0.38</td>
<td>0.38</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>1.21</td>
<td>1.26</td>
<td>1.22</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>1.00</td>
<td>1.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Nom. Int. Rates</strong></td>
<td>0.20</td>
<td>0.21</td>
<td>0.21</td>
<td>0.20</td>
<td>0.29</td>
<td>0.66</td>
<td>0.69</td>
<td>0.67</td>
<td>0.70</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>1.06</td>
<td>1.11</td>
<td>1.08</td>
<td>1.07</td>
<td>1.00</td>
<td>3.45</td>
<td>3.59</td>
<td>3.49</td>
<td>3.47</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>1.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Depreciat. Rate</strong></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.78</td>
<td>0.79</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.08</td>
<td>1.09</td>
<td>1.08</td>
<td>1.04</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Terms of Trade</strong></td>
<td>0.66</td>
<td>0.61</td>
<td>0.65</td>
<td>0.66</td>
<td>0.83</td>
<td>0.87</td>
<td>0.85</td>
<td>0.87</td>
<td>1.01</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>0.60</td>
<td>0.55</td>
<td>0.59</td>
<td>0.60</td>
<td>0.67</td>
<td>0.78</td>
<td>0.76</td>
<td>0.79</td>
<td>0.81</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>0.94</td>
<td>0.97</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Govt Deficit</strong></td>
<td>0.00</td>
<td>0.96</td>
<td>0.95</td>
<td>0.63</td>
<td>1.75</td>
<td>0.00</td>
<td>1.06</td>
<td>0.96</td>
<td>1.76</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>1.00</td>
<td>1.83</td>
<td>1.79</td>
<td>1.09</td>
<td>0.99</td>
<td>1.00</td>
<td>2.02</td>
<td>1.68</td>
<td>0.99</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Public Debt</strong></td>
<td>0.00</td>
<td>12.48</td>
<td>2.96</td>
<td>0.88</td>
<td>106.75</td>
<td>0.00</td>
<td>10.43</td>
<td>2.71</td>
<td>1.01</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>1.00</td>
<td>1.88</td>
<td>2.30</td>
<td>0.98</td>
<td>0.99</td>
<td>1.00</td>
<td>1.57</td>
<td>2.11</td>
<td>1.12</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>4.22</td>
<td>3.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Net Foreign Asset</strong></td>
<td>1.00</td>
<td>3.81</td>
<td>0.89</td>
<td>0.99</td>
<td>42.17</td>
<td>1.09</td>
<td>3.20</td>
<td>1.03</td>
<td>1.08</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>0.67</td>
<td>1.35</td>
<td>0.59</td>
<td>0.67</td>
<td>0.99</td>
<td>0.73</td>
<td>1.14</td>
<td>0.69</td>
<td>0.73</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>4.29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Deficit-to-GDP</strong></td>
<td>0.88</td>
<td>1.46</td>
<td>1.48</td>
<td>0.89</td>
<td>1.76</td>
<td>1.01</td>
<td>1.75</td>
<td>1.78</td>
<td>1.02</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>0.98</td>
<td>1.39</td>
<td>1.42</td>
<td>0.98</td>
<td>0.90</td>
<td>1.12</td>
<td>1.67</td>
<td>1.70</td>
<td>1.12</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>4.29</td>
<td>3.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Debt-to-GDP</strong></td>
<td>0.88</td>
<td>12.46</td>
<td>3.06</td>
<td>0.00</td>
<td>106.60</td>
<td>1.01</td>
<td>10.42</td>
<td>2.88</td>
<td>0.00</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>0.98</td>
<td>1.86</td>
<td>1.95</td>
<td>1.00</td>
<td>0.99</td>
<td>1.12</td>
<td>1.55</td>
<td>1.83</td>
<td>1.00</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>4.07</td>
<td>3.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>NFA-to-GDP</strong></td>
<td>1.27</td>
<td>3.93</td>
<td>1.19</td>
<td>1.27</td>
<td>42.38</td>
<td>1.38</td>
<td>3.35</td>
<td>1.34</td>
<td>1.39</td>
</tr>
<tr>
<td>Ratio to Wickselian</td>
<td>0.79</td>
<td>1.36</td>
<td>0.74</td>
<td>0.79</td>
<td>0.99</td>
<td>0.86</td>
<td>1.16</td>
<td>0.83</td>
<td>0.86</td>
</tr>
<tr>
<td>Ratio to &quot;High&quot;</td>
<td>3.31</td>
<td>2.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Entries are standard deviations in %.
Country F follows the Wicksellian Monetary Rule and the Deficit Feedback Rule