Ambiguity aversion and rollover risk: a possible explanation for market freezes?

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Abstract

In this paper, we discuss the collective role of rollover risk, ambiguity, ambiguity aversion, and a specific set of behavioral traits in determining freezes in the market for secured borrowing.

Under these circumstances, the market can experience a freeze since the evaluation of any random payoff is affected by ambiguity and ambiguity aversion, so that the debt capacity of a risky asset might go to zero as the number of debt-rollovers infinitely increases.

Specifically, what is crucial is the way agents perceive the familiar current state of the economy. If the economy is perceived as sick, so that "nothing can be worse than this", then a freeze may occur, but only for extremely high ambiguity levels. Vice versa, if the familiar state is not too detrimental for the economy, or better than some unknown situation, then the positive probability that the economy might experience an unfamiliar state will induce a desire for prudence, that directly impacts on the way agents evaluate future payoffs, and may lead to a freeze.

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†I have greatly benefited from the discussion with Professor Massimo Guidolin. All remaining errors are mine.
1 Introduction

In the context of the crisis that most financial markets are currently facing, it is of crucial importance on the one hand to develop a class of new models that can provide us with a better comprehension of the causes of the current situation, and, on the other hand, to identify decisive actions that will restore normal market functioning and investors' trust in the strength of key financial institutions and markets. Indeed, traditional models seem to be inappropriate to explain the economic and financial challenges we are facing. Similarly, classic tenets of central banking seem not to work well, since, notwithstanding the efforts of the various policy makers, the financial crisis appears to have intensified over the Summer 2008, acutely hitting companies such as the U.S. government-sponsored enterprises (GSEs) Fannie Mae and Freddie Mac, the investment bank Lehman Brothers, and the insurance company American International Group (AIG).

Specifically, following a growing body of research that views markets' complexity and the poor quality of the available information as the main causes of investors' behavior that is inconsistent with the predictions of standard models, the aim of this paper is the development of a simple theoretic framework that can explain freezes in the market for rolled-over secured borrowing, through ambiguity, ambiguity aversion, and the characterization of some specific behavioral traits of financial decision makers. In the simplified set-up we are going to consider, a market for rollover debt is said to experience a freeze when the debt capacity of the risky assets under consideration drops to zero.

The analysis of markets for secured borrowing is certainly crucial for a better understanding of the credit crunch. Indeed, financial researchers (see for example Diamond and Rajan [5]) generally agree that the crisis has its origin in the misallocation of resources to real estate, financed through the issuance of complicated financial products. These instruments were largely purchased by commercial and investment banks and financed through the issuance of short-term debt.

With the boom of the housing market, securitization became extremely
important to reduce the risk associated to mortgage loans, especially for international investors: if the mortgage was packaged together with mortgages from other areas, diversification would reduce the risk. As noticed by Diamond and Rajan [5], given the high demand for AAA paper, "securitization became focused on squeezing out the most AAA paper from an underlying package of mortgages; the lower quality securities issued against the initial package of mortgages were packaged together with similar securities from other packages, and a new range of securities, including a large quantity rated AAA, issued by this Collateralized Debt Obligation".

The repeated securitization process originated complicated exotic securities, whose evaluation was extremely complex, especially when house prices started to decline. Apparently bankers considered these securities attractive investments, underestimating their risk. But precisely because of the exaggerate riskiness of these instruments, investors would have demanded high premia for financing the bank long term, while, for short-term claims, the excessive risk would have been compensated by the option to forgo the investment earlier. Therefore, banks started issuing short-term debt without considering the possibility of becoming illiquid and unable to rollover the debt. On the contrary, expectations of future lower interest rates (supported for example by the so-called "Greenspan Put") further urged banks to become even more illiquid.

When house prices started falling, consequent mortgage defaults turned out to be seriously relevant. Hence, mortgage backed securities fell in value, and became impossible to be priced using standard techniques. They were hard to borrow against, even short term; as a result banks became illiquid, and unable to rollover financing. A concrete example of how this process has leaded to market freezes is the worldwide collapse in the market for Asset Backed Commercial Paper in the Summer 2007. Measures of market liquidity of these assets are difficult to obtain, nevertheless the decline in the amount of ABCPs has been estimated to have reached $3000bn between early August and early November in the U.S. market alone (see Appendix and [3]). Another relevant case of market breakdown is represented by Bear Stearns’ bankruptcy. In particular, as pointed out by the Security
and Exchange Commission’s Chairman Christopher Cox, the failure of Bear
Stearns seems to be more imputable to the bank’s inability to issue short
term debt backed by assets with relatively high credit risk rating, than to
the lack of capital, which has always been above the requirements of the
Basel II standards (See Appendix and [4]).

Our work is mainly inspired to a recent working paper by Acharya, Gale
and Yorulmazer, in which the debt capacity of a finitely lived asset is ana-
lyzed in a framework characterized by (i) short-term debt; (ii) risk of fire sale
in case of borrower’s default, and (iii) a "pessimistic" information structure.
Nevertheless, our model is only stimulated by [1], since it departs signif-
ically from it in terms of focus and provided results. In particular, [1]
shows that debt capacity depends on how information about the quality of
the asset is perceived (broadly speaking, if the arrival of no news about an
asset is perceived as good or bad news). In our model instead, we charac-
terize a theoretic framework and a collection of different behavioral traits that
can seriously impact on the way the market reacts to random events (and
hence on the valuation of the debt capacity of a risky asset, as well). Specif-
ically, we introduce ambiguity and ambiguity aversion, meaning that there
is a positive probability that the economy might experience some unfamil-
iar situation characterized by too vague an information level, that prevents
agents from being sufficiently confident in their probabilistic evaluations of
random future payoffs.

Traditional finance theory assumes that agents are either expected (EU)
or subjective expected (SEU) utility maximizers. That is, they choose
among alternative investment opportunities by simply confronting the re-
spective expected utility values, computed through a unique probability
distribution, which might be objectively given (EU) or subjectively derived
(SEU). Experimental works in finance and in decisions contradict both EU
and SEU predictions. In particular, one of the most popular evidence of
people’s systematic violation of (subjective) expected utility is described by
Ellsberg’s paradox [6], that provides a comparison of different attitudes of
the same agent when facing alternative sources of uncertainty.

Broadly speaking, Ellsberg’s experiment shows that people do not gener-
ally like situations in which they are not able to derive a unique probability distribution over the reference state space. These situations have became known as situations of *ambiguity*, and the general "dislike" for them as *ambiguity aversion*. This attitude cannot be reflected by SEU or EU models, since they do not allow agents to express their own degree of confidence about a probability distribution. In fact, under ambiguity, not only is the payoff deriving from the choice of an act uncertain, but also its expected value, since it can be evaluated using different probability distributions that are all plausible. Ambiguity aversion induces agents’ prudent behavior which is reflected in the functional that represents each agent’s preferences.

The emergence of decision theoretic models that are less narrow than (subjective) expected utility has induced the growth of new fields of research in which these models are applied in standard macroeconomic and finance contests, with the aim of achieving a better representation of reality.

In our framework, ambiguity and ambiguity aversion are introduced through Schmeidler’s representation (the so called CEU model) for ambiguity averse preferences [7].

In his seminal paper, Schmeidler notices that the probability attached to an uncertain event does not reflect the heuristic amount of information that led to the assignment of that probability. Motivated by this consideration, he suggests the assignment of non-additive probabilities, in order to allow for transmission or recording of information that additive probabilities cannot represent. The main novelty is that the CEU model also accounts for phenomena that do not occur when risk only is considered, as the violation of expected utility theory described by Ellsberg. Specifically, weakening the standard axioms of (subjective) expected utility, Schmeidler has developed a functional representation for preferences over random outcomes, that, through non-additive probabilities, allows for the fact that agents may not be fully confident on their probability assignment over uncertain events.

In general, the introduction of ambiguity and ambiguity aversion in financial models is highly justified. History of financial markets is in fact full of episodes of increase in uncertainty that have leaded to liquidity problems, especially in emerging markets. Further, pricing models’ uncertainty, result-
ing from the poor quality of information on which agents base the choice of their model, is acknowledged by most operators as a cause of serious mis-pricing errors, especially for derivative instruments. Finally, Bernanke in a recent speech at the Economic Club of New York has pointed out how many novel aspects that characterize the latest crisis of financial markets are largely arising from the complexity and the sophistication of the valuation techniques and financial instruments currently available, that result in considerable variation in fair value estimates, and valuation problems, in turns, translate into capital shortages.

At a theoretical level, our analysis suggests that ambiguity and ambiguity aversion can be responsible for sudden freezes in the markets for rollover debt, because they have large impact on the evaluation of the debt capacity of the risky asset under consideration.

Specifically, a crucial feature of our setting is that the uncertain state is not necessarily perceived as negative, in the sense that the actual probability of default of the asset is not conditioned by the realization of the unfamiliar state. Nevertheless, ambiguity and ambiguity aversion impact on the way agents make their evaluations. Under these circumstances, the market can experience a freeze, since the evaluation of any future random payoff is affected by ambiguity and ambiguity aversion, so that the debt capacity of a risky asset might go to zero as the number of debt-rollovers infinitely increases.

Ambiguity and ambiguity aversion are per se sufficient in determining the freeze, and no liquidation cost, or fire sale is needed to derive our result. What is crucial, instead, is agents’ perception of the familiar current state of the economy. If the economy is perceived as sick, so that "nothing can be worse than this", then the freeze may occur, but only under some particular conditions that can be interpreted as extremely high ambiguity levels. Vice versa, if the current state is considered as neutral, or anyway better than some unknown situation, then a positive probability that the economy would experience an unfamiliar state induces a desire for prudence, that directly impacts on the way agents evaluate future payoffs, and can lead to a market freeze.
Finally, we want to point out that ours is not the first model that tries to generate market freezes through ambiguity. A recent work of Caballero and Krishnamurthy [2] introduces Gilboa and Schmeidler max-min preferences to develop a model of financial crises and central bank policy under ambiguity and ambiguity aversion. Specifically, Caballero and Krishnamurthy identify flight to quality episodes as a main cause for financial instability, and they show that, when the aggregate quantity of liquidity is limited, ambiguity averse agents fear that there won’t be enough liquidity available in case they suddenly need it. Under this circumstances, agents’ willingness to make risky investments is reduced, and capital is moved away towards the safest possible vehicles.

As [2], our paper tries to explain financial crises through ambiguity and ambiguity aversion. However, the focus of the analysis is different. Caballero and Krishnamurthy show how flight to quality episodes might occur when the economy is currently experiencing a state of uncertainty. In our model, instead, what is crucial in determining the market freeze is the possibility (that is, a positive probability) that the economy will experience an uncertain situation.

Further, in [2], ambiguity impacts each agent’s evaluation in a market perspective, in the sense that investors fear that their shock will occur late relative to others. In our model, instead, the perspective is purely individual, since ambiguity and ambiguity aversion directly affects the evaluation of assets’ payoffs.

The paper proceeds as follows: Section 2 characterizes the decision theoretical framework and the economic set up. Section 3 explicitly solves the model. Section 4 concludes. All proofs are collected in the Appendix.

2 Setting

In this Section we characterize our framework. Specifically, we briefly recall Schmeidler’s model for ambiguity averse preferences, and we describe the economic-financial setting. Finally, we show how the CEU model fits in our set up.
2.1 The CEU model

To allow for ambiguity and ambiguity aversion, maintaining tractability, we use Schmeidler’s representation (henceforth "CEU model") for ambiguity averse preferences (see [7] for further details).

In his seminal paper, Schmeidler criticizes the standard paradigm of expected utility according to which, if the set of possible states of the word is made of $k$ equiprobable states, then the probability of each state is $1/k$, so that the sum of these probabilities adds up to 1. Specifically, he notices that the probability attached to an uncertain event does not reflect the heuristic amount of information that led to the assignment of that probability. For example, when there are only two possible equiprobable events, they are assigned probability $1/2$ each, independently of whether the available information is meager or abundant.

Motivated by this consideration, he suggests the assignment of non-additive probabilities (meaning that they not add up to 1), or capacities, in order to allow for transmission or recording of information that additive probabilities cannot represent. To measure the amount of available information (and consequently the ambiguity level), Schmeidler proposes the index:

$$A(v) = 1 - \sum_{s_k \in S} v(s_k)$$

where each $v(s_k)$ is the specific non-additive probability (capacity) assigned to the event $s_k$ in the state space $S$. A lower index $A(v)$ indicates more precise information, and, therefore, lower ambiguity. In particular, $A(v) = 0$ corresponds to the subjective expected utility case, without ambiguity, and with additive probabilities.

In the context of the previous example, when the outcomes of a gamble are mutually exclusive, and conditioned on some event $B$, the number $A(v) = 1 - v(B) - v(B^c)$ would indicate the decision maker’s confidence in the non-additive probability assessment $(v(B), v(B^c))$.

The main novelty is that Schmeidler’s model accounts also for phenomena that do not occur when only risk is considered, as the violation of
expected utility theory described by Ellsberg. Specifically, weakening the standard axioms of (subjective) expected utility, Schmeidler develops a functional representation for preferences over random outcomes, that, through non-additive probabilities, allows for the fact that agents’ may not be fully confident on their probability assignment over uncertain events. In particular, given a capacity over the reference state space, expectations are computed through the Choquet integral.

Formally, assuming that a gamble has only two possible outcomes, say $H$ or $L$, a capacity (or non-additive probability) $v$ is any assignment to the events "neither $H$ or $L$ occur", "$H$ or $L$ or both occurs", "$H$ and $L$ both occur", "$H$ occurs", "$L$ occurs", such that:

1. $v(\text{neither } H \text{ nor } L \text{ occur}) = 0$
2. $v(H \text{ or } L \text{ or both occurs}) = 1$
3. $v(H), v(L) \geq 0$
4. $v(H) + v(L) \leq 1$

Finally, letting $u$ be a standard von-Neumann Morgenstern utility function, the expected utility of the gamble is evaluated through the Choquet integral as:

$$\min_{\mu \in C(v)} [\mu \times u(H) + (1 - \mu) \times u(L)]$$

$$C(v) = \{\mu \in [0, 1], \ \mu \geq v(H), 1 - \mu \geq v(L)\}$$

the ambiguity level associated to $v$ is given by $A(v) = 1 - v(H) - v(L)$. $C(v)$ is the core of the capacity $v$, that is, the set of additive probability distributions that eventwise dominate $v$. $C(v)$ is interpreted as the set of effective priors considered by the agents, and ambiguity is reflected by its multivalued nature. Decision makers express ambiguity aversion by assigning higher probabilities to unfavorable states, as reflected in the $\min$ operator.

After Schmeidler’s seminal paper, a growing body of the decision theoretic literature has been dedicated to ambiguity and ambiguity aversion,
and many criticisms to the CEU model have been suggested. Nevertheless, our choice for the CEU representation is justified by the tractability of the model, that results in being suitable for the financial problem under consideration.

2.2 The financial market

A structured investment vehicle (SIV) is set up at time $0$. The SIV holds a collateral asset with maturity $1$, and terminal value that can be either $0$, in case of default (say event $L$), or $V > 0$, in case of success (say event $H$). For simplicity, the current yield is set to zero.

We do not assume the existence of any friction, such as information asymmetries, transaction, and/or liquidation costs.

The SIV has to raise asset-backed finance, by issuing short term debt with fixed maturity $0 < \tau < 1$. Hence, the debt is rolled over $N$ times, where $N$ is such that: $(N + 1)\tau = 1$. Let us denote by $t_n = n\tau$, $n = 0, 1, ..., N + 1$, the date at which the $n$-th rollover occurs.

2.3 Ambiguity

The economy starts from some initial familiar situation at time $0$ (not necessarily considered as favorable to the financial market), say $s_0 = F$. As time passes, the economy can stay into the familiar state $F$ or move to an unfamiliar one $U$, due to some random event, that occurs with probability $1 - q$. We denote by $s_n$, $s_n = F, U$, the generic state that occurs at time $t_n$.

In the uncertain state, the market is not able to derive a unique prior over the possible future evolution of the economy (namely if it will stay into the unfamiliar state or switchback to the familiar one). Therefore, since agents are ambiguity averse, if the unfamiliar state occurs at time $t_n$, the two next-period possible contingent states $s_{n+1} = U$ and $s_{n+1} = F$ are respectively assigned capacities $v(U)$ and $v(F)$.

Consequently, the transition between dates $t_n$ and $t_{n+1}$ are assumed to
be governed by a stationary capacity matrix $M$:

$$M = \begin{bmatrix} q & 1 - q \\ v(F) & v(U) \end{bmatrix}$$

where, $v(\cdot)$ is a capacity associated to the event "the economy moves from the unfamiliar state to state (\cdot)", and $q$ is the probability of staying in the familiar state at time $t_{n+1}$, given the fact that the familiar state has already occurred at time $t_n$.

Consequently, any time $t_{n+1} \neq t_{N+1}$ contingent payoff $x = \{x(F), x(U)\}$ is evaluated at time $t_n$ as:

$$E_{t_n} \left[ x \mid s_n = F \right] = qx(F) + (1 - q)x(U) \quad (1)$$

$$E_{t_n} \left[ x \mid s_n = U \right] = \min_{(\mu, 1 - \mu) \in C(v)} \{\mu x(F) + (1 - \mu) x(U)\} \quad (2)$$

where

$$C(v) = \{ (\mu, 1 - \mu) : \mu \in [0, 1] \quad \mu \geq v(F), 1 - \mu \geq v(U) \}$$

Similarly, at the penultimate date $t_N$, if the economy is in state $s_N = U$, the market is not able to derive a unique probability distribution for the two events "success" ($H$), and "default" ($L$) of the asset. Therefore, it assigns to the occurrences $H$ and $L$, capacities $w(H)$ and $w(L)$, respectively, and, again, evaluates random (uncertain) payoffs, according to the CEU model. Viceversa, in the familiar state $s_N = F$, the market sets the probability of success to $p$, and to $1 - p$ the one of default. Hence, any time $t_{N+1}$ contingent payoff $x = \{x(H), x(L)\}$ is evaluated at time $t_N$ as:

$$E_{t_N} \left[ x \mid s_N = F \right] = px(H) + (1 - p)x(L)$$

$$E_{t_N} \left[ x \mid s_N = U \right] = \min_{(\mu, 1 - \mu) \in C(w)} \{\mu x(H) + (1 - \mu) x(L)\}$$

where

$$C(w) = \{ (\mu, 1 - \mu) : \mu \in [0, 1] \quad \mu \geq w(H), 1 - \mu \geq w(L) \}$$
2.3.1 Behavioral assumptions

To characterize the capacities \( v(\cdot) \) and \( w(\cdot) \), we specify a collection of behavioral traits that describe the agents that populate the economy.

Specifically:

b.t.1 If the economy is perceived as healthy, the increase in uncertainty that characterizes the unfamiliar state is regarded as a danger, so agents choose \( w(H) \), such that \( p \geq w(H) \).

b.t.2 If the economy is extremely sick, agents choose \( w(H) \), such that \( p < w(H) \).

b.t.3 States of the economy tend more to be persistent over time than to change, hence:

\[
q \geq 1 - q \\
q \geq v(F)
\]

Note that assuming \( q \geq v(F) \) does not imply that agents believe that \( pr(s_n = F | s_{n-1} = F) > pr(s_n = F | s_{n-1} = U) \). Indeed, \( v \) is not the effective probability that the financial market is going to consider in its evaluations. In practice, all the probability distributions that eventwise dominate \( v \), will be considered.

To see why b.t.3 implies \( q \geq v(F) \), suppose instead that \( q < v(F) \) holds. Hence, the set of effective priors considered by the agents is made of all probability distributions \( (\mu(F), \mu(U)) \), such that \( \mu(F) \geq v(F) > q \).

Moreover, \( \mu(U) = 1 - \mu(F) < 1 - q \), and, since \( q \geq 1 - q \), it thus follows that \( \mu(F) > (q \geq) \mu(U) \), which contradicts b.t.3.

\footnote{Note that the requirement \( p < w(H) \) precisely reflects the feeling that "nothing can be worse than the current situation". Indeed, according to the CEU model, agents consider all the priors that eventwise dominate \( w \), that is, all distributions according to which the probability of success is greater than \( w \), and, therefore, than \( p \).}

\footnote{To preserve generality, we maintain weak all the inequalities, even if at least one has to be strengthened to not incur into triviality.}
Notice instead, that we are not assuming \( v(U) \geq 1 - q \), since such a requirement could be misleading, and even lead to a perverse behavior, by inducing a forced over-evaluation of the probability of staying in the unfamiliar state, with consequent over-evaluation of those state-contingent payoffs with highest realization in the unfamiliar state.\(^3\) Finally, such an assignment would reflect a fake reduction in the ambiguity-ambiguity aversion level, as can be easily seen by observing that the index \( A(v) \) would be artificially reduced, without any actual increase in the available information.

If agents are characterized by b.t.1, we say that the economy is in the optimistic framework; in the pessimistic one, if instead b.t.2 holds.

The following assumption on the available information has the only aim to simplify the analysis (especially notation), without reducing generality.\(^4\)

**Information Assumption (I)** Under the optimistic framework, in the unfamiliar state there is no information that signals that the realization of one state will be more likely than the other, hence \( v(F) = v(U) = v \).

Lemma 1 reports a result that will be useful in the subsequent analysis:

**Lemma 1** Consider a time \( t_n \) contingent payoff \( x = \{x(U), x(F)\} \), and denote its evaluation at time \( t_n \), by \( E_{t_n} [x | s_n = F] \), if the familiar state realizes at time \( t_n \), and by \( E_{t_n} [x | s_n = U] \), otherwise.

Then the following implications hold:

1. If there exists \( n^* \), \( 1 \leq n^* \leq N \), such that:

   \[
   E_{t_n^*} [x | s_{n^*} = U] \leq E_{t_n^*} [x | s_{n^*} = F]
   \]

   then for any \( n < n^* \),

   \[
   E_{t_n} [x | s_n = U] < E_{t_n} [x | s_n = F]
   \]

\(^3\)Notice that, since at each time \( t_n \), the events "the familiar state realizes" and "the unfamiliar state realizes" are mutually exclusive, because of the specific formal characterization of Schmeidler’s functional representation, the evaluation of a state-contingent payoff with highest realization in the familiar state is not affected by the numerical value of \( v(U) \).

\(^4\)Specifically, the I-assumption is used only in the optimistic framework to maintain notation as plain as possible, and to derive some comparative statics results concerning different degrees of ambiguity aversion. General results do not rely on the assumption.
2. Given the condition specified in 1,

\[ E_{t_n} [x \mid s_n = U] = v(F) E_{t_{n+1}} [x \mid s_{n+1} = F] + \ldots \]
\[ \ldots + (1 - v(F)) E_{t_{n+1}} [x \mid s_{n+1} = F] \]

The first part of the Lemma states that if at any rollover date \( t_n \) the occurrence of the unfamiliar state (weakly) hurts valuation, then the unfamiliar state (strictly) reduces valuation at all times that precede \( t_n \). Further, according to the second part, if the unfamiliar state realizes at any date before \( t_n \), then the effective probability considered by the agents at that date is the one that assigns probability \( v(F) \) to the familiar state, and probability \( 1 - v(F) \) to the unfamiliar one.

3 The model

For simplicity, let us assume that the market is risk neutral, and that the riskfree rate is zero.

As usual in banking theory, we define the debt (or borrowing) capacity of a risky asset to be the maximum amount that can be borrowed, using only the asset as collateral.

For any \( t_n \), we denote by \( B^F_n \) the debt capacity of the asset evaluated at time \( t_n \), if \( s_n = F \). Similarly, \( B^U_n \) is the debt capacity of the asset evaluated at time \( t_n \), if \( s_n = U \). Hence the borrowing capacity, as a function of the current state, is given by:

\[ B^C_n = \max_D E_{t_n} [D \mid s_n = (\cdot)] \]

where \( D \) is the face value of the debt issued at time \( t_n \).

A freeze in the market for short term borrowing occurs if the debt capacity of the asset drops to zero.

What is crucial in our framework is the way expectations are derived. Specifically, if \( s_n = F \), standard rules apply, and agents simply compute expected values by averaging contingent payoffs with probability weights \( q \)
and $1 - q$. Viceversa, if $s_n = U$, then expectations are evaluated through the Choquet integral.

### 3.1 The optimistic framework \( p \geq w(H) \)

#### 3.1.1 General characterization

Let us start with the analysis under the optimistic framework. As stated in b.t.1, this corresponds to the assignment of a capacity \( w(H) \leq p \) to the event of success.

At the penultimate date \( t_N \), the economy can be either in the familiar or the unfamiliar state. In both cases, the face value of the debt issued at \( t_N \), \( D \), cannot be larger than \( V \), otherwise the SIV will default for sure, since rational agents would never subscribe such a debt contract.

Hence, the maximum amount that can be borrowed by the SIV at time \( t_N \), that is, its borrowing capacity at \( t_N \),\(^5\) is:

\[
B_F^N = pV \\
B_U^N = w(H)V
\]

In the optimistic framework, \( B_F^N > B_U^N \).

Going back to \( t_{N-1} \), the economy can be either in the familiar or the unfamiliar state, in both cases, to avoid default, the face value of the debt issued at \( t_{N-1} \), \( D \), cannot be larger than \( B_F^N \). Hence, using (2) and (1), the evaluation of \( D \) at time \( t_{N-1} \) is:

\[
E_{t_{N-1}}[D|s_{N-1} = F] = \begin{cases} 
(1 - q) B_U^N + qD & B_U^N < D \leq B_F^N \\
D & B_U^N \leq D
\end{cases}
\]

\[
E_{t_{N-1}}[D|s_{N-1} = U] = \begin{cases} 
(1 - v) B_U^N + vD & B_U^N < D \leq B_F^N \\
D & B_U^N \leq D
\end{cases}
\]

\(^5\)Note that at the penultimate date, the borrowing capacity coincides with the evaluation at \( t_N \) of the maximal face value of the debt.
By definition, $B_{N-1}^F = \max_{D \leq B_N^F} E_{t_{N-1}} [D | s_{N-1} = (\cdot)]$, therefore:

$$B_{N-1}^F = \max \{ B_N^U, (1 - q) B_N^U + q B_N^F \}$$

$$B_{N-1}^U = \max \{ B_N^U, (1 - v) B_N^U + v B_N^F \}$$

Notice that $B_{N-1}^F > B_{N-1}^U$.

Usual iteration methods lead to the following characterization of the path of the borrowing capacity:

**Theorem 2** At the penultimate date, $t_N$, the borrowing capacity can be either $pV$ or $w(H)V$, depending on whether the economy is in the familiar or the unfamiliar state.

For any $n < N$, the conditional borrowing capacity is:

$$B_{n-1}^F = \max \{ B_n^U, (1 - q) B_n^U + q B_n^F \} \quad (3)$$

$$B_{n-1}^U = \max \{ B_n^U, (1 - v) B_n^U + v B_n^F \} \quad (4)$$

depending on the realization of the state $s_n$.

A further description of the paths of the conditional borrowing capacities under the optimistic framework is provided by the following Proposition.

**Proposition 3** For any $n \leq N$, the conditional debt capacities satisfy:

1. $B_n^U < B_n^F$;
2. $B_{n-1}^F < B_n^F$;
3. $B_{n-1}^U \geq B_n^U$, in particular, if there exists $n^*$, such that $B_{n^*-1}^U = B_{n^*}^U$, then $B_n^U = B_{n^*}^U$ for any $n \leq n^*$.

The first inequality derived in Proposition 3 assures that, under the optimistic framework, at any rollover date $t_n$, the debt capacity is higher if the familiar state realizes.
The second inequality has intuitive meaning: as the time to maturity of the asset approaches, the borrowing capacity evaluated in the familiar state increases, because the probability of not incurring into the unfamiliar state $q^{N-t_n}$ also increases, and this has a positive effects on the evaluation by part 1 of the Proposition.

A similar argument applies to the last inequality: as the time to maturity of the asset approaches, the minimal probability of not incurring into the familiar state, $(v(U))^{N-t_n}$, increases, reducing the evaluation of the borrowing capacity of the asset, again by part 1.\footnote{Notice that, as the number of rollovers infinitely increases, the existence of $n^*$ as required in Proposition 3 is more likely, since $B_n^F$ and $B_{n}^F$ move in opposite directions as $n$ reduces, yet $B_{n}^U < B_{n}^F$.}

Finally, the effect of an increase in uncertainty are characterized in Proposition 4. Specifically, two different financial markets that are identical to the one described so far are considered. The only difference is that one of them is characterized by lower ambiguity, so that, instead of the capacity $v$, evaluations are based on the capacity $\tilde{v} > v$,\footnote{Notice that here we are using Assumption I to establish a comparative order between different degrees of ambiguity aversion.} so that $A(\tilde{v}) < A(v)$. Indeed, according to the CEU model, the ambiguity level that characterizes each of the two markets can be represented through the index:

$$A(\mu) = 1 - \mu(U) - \mu(F)$$
$$A((\mu(U), \mu(F)) = (v, v), (\tilde{v}, \tilde{v})$$

hence

$$\tilde{v} > v \Rightarrow A(\tilde{v}) = 1 - 2\tilde{v} < 1 - 2v = A(v)$$

Let us denote by $B_n^{Fi}, B_n^{Ui}, i = \tilde{v}, v$, the state dependent borrowing capacities that characterize the two financial markets at time $t_n$.

**Proposition 4** Suppose $\tilde{v} > v$, then the following properties hold for any $n < N$:

1. $|B_n^{U\tilde{v}} - B_n^{F\tilde{v}}| < |B_n^{Uv} - B_n^{Fv}|$;
2. \(|B_{n-1}^{F\tilde{v}} - B_n^{F\tilde{v}}| < |B_{n-1}^{Fv} - B_n^{Fv}|;\)

In particular, property 2 has the following important underlying intuition: since the ambiguity problem is perceived as less severe in the market characterized by capacity \(\tilde{v}\), the occurrence of the unfamiliar state is considered less detrimental than in the other one. Hence, as the expiration date approaches, the marginal benefit provided by the increase in the probability of not incurring into the unfamiliar state is lower.\(^9\)

### 3.1.2 The "freeze-result"

The optimistic framework guarantees that we can use Lemma 1 (since \(pV \geq wV\)) to derive property 1 in Proposition 3. Hence, the condition required by Lemma 1 part 2 holds true, and the financial problem under uncertainty can be mathematically treated as a normal evaluation in a setting characterized by transition matrix \(M\):

\[
M = \begin{bmatrix}
q & 1 - q \\
v(F) & 1 - v(F)
\end{bmatrix}
\]

Hence, as in [1], the effect of an increase in the number of rollovers can be easily analyzed by reducing the length of each time interval, holding constant the probability of switching states in each unit of time. Specifically, we choose \(\alpha, \beta > 0\) such that

\[
1 - v(\tau) = e^{-\alpha \tau} \quad q(\tau) = e^{-\beta \tau}
\]
satisfy b.t.3, where \(\tau\) is the period length. As usual, let \(B^i_n(\tau)\) be the borrowing capacity at date \(t_n\), if \(s_n = i, i = F, U\).

Theorem 5 assures that \(B^F_n(\tau)\) is bounded below away from 0, even if the number of rollovers becomes infinite. Hence, it is always positive, no matter how small \(\tau\) can be.

\(^9\)Such a probability is \(q^{N-t_n}\) in both markets.
Theorem 5

1. \( \forall n, \tau, \)
   \[ B_n^F(\tau) \geq q(\tau)^{N-n} \hat{V} \]
   \[ \text{where } \hat{V} = (1-q)w(H)V + qpV, \; \tau = \frac{1}{N} \]

2. As \( \tau \to 0 \) and \( n\tau \to t \), \( B_n^F(\tau) \geq e^{-\beta(1-t)} \hat{V} \)

Theorem 6

As \( \tau \to 0 \), \( B_1^U(\tau) \to 0 \)

3.2 The pessimistic framework \( p < w(H) \)

3.2.1 General characterization

Next we consider the pessimistic framework. As stated in b.t.2, this corresponds to the assignment of a capacity \( w(H) > p \) to the event of success. This framework can be thought of as a situation in which the economy is consider so sick, that a change in the current situation can be perceived as beneficial. Hence, the main difference with the optimistic setting is that the realization of the uncertain state is not considered detrimental for the financial market. Nevertheless, to avoid overconfidence into a completely uncertain environment, the financial market still evaluates uncertain payoffs through the CEU model.

We repeat the analysis of the previous setting. Again, at the penultimate date \( t_N \), the economy can be either in the familiar or the unfamiliar state. In both cases, the face value of the debt issued at \( t_N \), \( D \), cannot be larger than \( V \), otherwise the SIV will default for sure. Hence, the maximum amount that can be borrowed by the SIV at time \( t_N \) is:

\[ B_N^F = pV \]
\[ B_N^U = w(H)V \]

Notice that now we have \( B_N^F < B_N^U \).
Going back to $t_{N-1}$, the economy can be either in the familiar or the unfamiliar state, in both cases, to avoid default, the face value of the debt issued at $t_{N-1}$, $D$, cannot be larger than $B^U_N$. Hence, using (2) and (1), the evaluation of $D$ at time $t_{N-1}$ is as usual:

$$E_{t_{N-1}}\left[D | s_{N-1} = F\right] = \begin{cases} (1-q)B^U_N + qD & B^F_N < D \leq B^U_N \\ D & B^F_N \leq D \end{cases}$$

$$E_{t_{N-1}}\left[D | s_{N-1} = U\right] = \begin{cases} (1-v(U))B^F_N + v(U)D & B^F_N < D \leq B^U_N \\ D & B^F_N \leq D \end{cases}$$

Using the definition of $B^{(i)}_{N-1}$, we get:

$$B^F_{N-1} = \max \left\{ B^F_N, (1-q)B^U_N + qB^F_N \right\}$$

$$B^U_{N-1} = \max \left\{ B^F_N, (1-v(U))B^F_N + v(U)B^U_N \right\}$$

The problem is that now we are no longer able to characterize the relation between $B^F_{N-1}$ and $B^U_{N-1}$, since the behavioral traits described above do not provide any useful bound for $v(U)$, unless under further misleading restrictions. Specifically, the assumption $v(U) \geq 1-q$ would imply $B^F_{N-1} < B^U_{N-1}$, allowing for a recursive formulation of the problem. Nevertheless, as discussed above, the inequality $v(U) \geq 1-q$ has no behavioral foundation, therefore, in general, it is not possible to proceed recursively as in the previous Subsection.

In general, we can only characterize the paths of the conditional borrowing capacities as we did in Theorem 1, whose analog in the pessimistic framework is the following.

**Theorem 7** At the penultimate date, $t_N$, the borrowing capacity can be either $pV$ or $w(H)V$, depending on whether the economy is in the familiar or the unfamiliar state.

At time $t_{N-1}$, the conditional borrowing capacity is:

$$B^F_{N-1} = \max \left\{ B^F_N, (1-q)B^U_N + qB^F_N \right\}$$

$$B^U_{N-1} = \max \left\{ B^F_N, (1-v(U))B^F_N + v(U)B^U_N \right\}$$
For any \( n < N - 1 \), the conditional borrowing capacity is:

\[
B_{n-1}^F = \max \left\{ B_{n}^{\min}, (1 - q) B_{n}^U + q B_{n}^F \right\}
\]

\[
B_{n-1}^U = \max \left\{ B_{n}^{\min}, \min_{(\mu, 1-\mu) \in C(v)} \mu B_{n}^U + (1 - \mu) B_{n}^F \right\}
\]

depending on the realization of the state \( s_n \) where

\[
B_{n}^{\min} = \min \left\{ B_{n}^U, B_{n}^F \right\}
\]

\[
C(v) = \{(\mu, 1-\mu) : \mu \in [0, 1], \mu \geq v(U), 1 - \mu \geq v(F)\}
\]

3.2.2 The "freeze-result" under high ambiguity

It could be of interest to point out that, if the ambiguity problem is extremely severe, the two capacities \( v(U) \) and \( v(F) \) will be very low in their values, as it is clearly reflected by the index \( A(v) = 1 - v(U) - v(F) \). In particular, holding \( v(F) \) fixed, higher degrees of ambiguity correspond to lower values of \( v(U) \).

If \( v(U) < 1 - q \), it can be shown that \( B_{N-1}^F > B_{N-1}^U \), and the entire analysis of the previous Section will apply, since the recursive arguments used above do not rely on the initial (for the procedure, final for the financial problem of reference) point. This is equivalent as to say that, given the objective probability distribution \((q, (1-q))\), if a market is characterized by capacities \( v(F) \) and \( v(U) = (1 - q) \), then any financial market characterized by same \( v(F) \), but higher ambiguity aversion will experience a freeze.

Hence, under these circumstances, even if agents believe that nothing can be worse than the familiar state, the market ends up in a freeze, as it is formalized in the following proposition.

**Proposition 8** Suppose \( wV > pV \) and \( v(U) < (1 - q) \) then:

1. At the penultimate date \( t_N \), the borrowing capacity can be either \( pV \) or \( w(H)V \), depending on whether the economy is in the familiar or the unfamiliar state.
At time $t_{N-1}$, the conditional borrowing capacity is:

$$
B_{N-1}^F = \max \{ B_N^F, (1 - q) B_N^U + q B_N^F \}
$$

$$
B_{N-1}^U = \max \{ B_N^F, (1 - v(U)) B_N^F + v(U) B_N^U \}
$$

For any $n < N - 1$ the conditional borrowing capacity is:

$$
B_{n-1}^F = \max \{ B_n^U, (1 - q) B_n^U + q B_n^F \}
$$

$$
B_{n-1}^U = \max \{ B_n^U, (1 - v) B_n^U + v B_n^F \}
$$

depending on the realization of the state $s_n$.

2. $B_n^U < B_n^F$, $B_{n-1}^F < B_n^F$ and $B_{n-1}^U \geq B_n^U$. Further, if there exists $n^*$, such that $B_{n^*-1}^U = B_{n^*}^U$, then $B_n^U = B_n^U$ for any $n \leq n^*$.

3. Theorem 5 and Theorem 6 hold.

At first sight, the inequality $v(U) < (1 - q)$ might be misleading, and let our last freeze result appear counterintuitive. Instead, there is no contradiction in Proposition 8, since $v(U)$ is not the effective probability assigned to the event "the economy will stay into the unfamiliar state also in the next period". Hence, for any fixed $v(F)$, lower values of $v(U)$ do not reflect more optimistic expectations on the possible evolutions of the economy, but instead they represent higher ambiguity since the set of effective probability distributions to be considered (the core of $v$) is wider.

3.3 Policy Implications

In the optimistic framework of our model, two different aspects worry the potential subscribers of the short term debts issued by the SIV. On the one side, there is the risk that the collateral asset might default, but this can be solved by shortening the length of the contract. Most importantly, agents fear that, if the unfamiliar state realizes, the SIV might suddenly become illiquid and incapable to pay the debt back. A central bank that credibly conveys the message that it commits itself to ensure stability of
the financial system, if necessary, through recapitalization of entities that have a possibility of survival, and the merger of those that do not, would address precisely this the latter concern. Specifically, agents would perceive this as a greater probability of restoring the familiar state, if the unfamiliar one occurs. Hence they will increase the value of the capacity \( v(F) \). In particular, provided that the message is strong enough, \( v(F) \geq q \). In our model this is what it is needed to prevent Lemma 1, part 1, to hold, which is the basis of our freeze result. In particular, it would be no longer true that if \( B_n^U < B_n^F \) at any date \( t_n \), then \( B_m^U < B_m^F \) at all preceding dates \( t_m < t_n \), as shown in the following Lemma.

**Lemma 9** Consider a time \( t_N \) contingent payoff \( x = \{ x(U), x(F) \} \), and denote its evaluation at time \( t_n \), by \( E_{t_n} [x|s_n = F] \), if the familiar state realizes at time \( t_n \), and by \( E_{t_n} [x|s_n = U] \), otherwise. If there exists \( n^* \), \( 1 \leq n^* \leq N \), such that:

\[
E_{t_n^*} [x|s_{n^*} = U] \leq E_{t_n^*} [x|s_{n^*} = F]
\]

then for any \( n < n^* \),

\[
E_{t_n} [x|s_n = U] \geq E_{t_n} [x|s_n = F] \quad n = n^* - (2i - 1)
\]

\[
E_{t_n} [x|s_n = U] \leq E_{t_n} [x|s_n = F] \quad n = n^* - 2i
\]

for all \( i \in N \) such that \( 1 \leq i \leq \frac{n^*}{2} \)

### 4 Conclusion

In this paper, we have discussed how rollover risk, ambiguity and ambiguity aversion, together with a collection of behavioral traits, that includes the way agents perceive the current familiar state of the economy with respect to an uncertain one, can contribute in determining a freeze in the market for secured borrowing.

\[\text{As noticed before this would imply that b.t.3 holds only when the economy is in the familiar state.}\]
Ambiguity implies that there is a positive probability that the economy would experience some unfamiliar situation, characterized by too vague an information level, that prevents agents from being sufficiently confident in their probabilistic evaluations of random future payoffs. Under these circumstances, the market can experience a freeze, since the evaluation of any future random payoff is affected by ambiguity and ambiguity aversion, so that the debt capacity of the collateral risky asset might go to zero, as the number of debt-rollovers infinitely increases.

Ambiguity and ambiguity aversion are per se sufficient in determining the freeze, and no liquidation cost or fire sale is needed to derive our result. What is crucial, instead, is the agents’ perception of the familiar current state of the economy. If the economy is perceived as sick, so that "nothing can be worse than this", then the freeze may occur, but only under some specific circumstances that can be interpreted as extremely high ambiguity levels. Viceversa, if the current state is considered as neutral, or better than some unknown situation, then the possibility that the economy would experience an unfamiliar state induces a desire for prudence, that directly impacts on the way agents evaluate future payoffs, and can lead to the market freeze.

Despite its simplicity, our model applies to a variety of financial settings that includes all markets characterized by a maturity mismatch between assets and liabilities.

We believe that the role of ambiguity and ambiguity aversion in the current financial crisis has clear empirical evidence, and hence should be further investigated. This is also the aim of our future research.
Appendix

Proof of Lemma 1. Let us start with the first implication.
Suppose \( \exists n^*, 1 \leq n^* \leq N \), such that \( E_{t_n^*} [ x | s_{n^*} = U ] \leq E_{t_{n^*}} [ x | s_{n^*} = F ] \), hence:

\[
E_{t_{n^*}} [ x | s_{n^*} = F ] = q E_{t_{n^*}} [ x | s_{n^*} = F ] + (1 - q) E_{t_{n^*}} [ x | s_{n^*} = U ]
\]
and

\[
E_{t_{n^*}} [ x | s_{n^*} = U ] = \ldots
\]

\[
\ldots = \min_{(\mu, 1 - \mu) \in C(v)} \{ \mu E_{t_{n^*}} [ x | s_{n^*} = F ] + (1 - \mu) E_{t_{n^*}} [ x | s_{n^*} = U ] \}
\]

Given (2), and \( E_{t_{n^*}} [ x | s_{n^*} = U ] \leq E_{t_{n^*}} [ x | s_{n^*} = F ] \)

\[
E_{t_{n^*}} [ x | s_{n^*} = U ] = v ( F ) E_{t_{n^*}} [ x | s_{n^*} = F ] + \ldots
\]

\[
\ldots + (1 - v ( F )) E_{t_{n^*}} [ x | s_{n^*} = U ]
\]

Finally, by b.t.3, \( E_{t_{n^*}} [ x | s_{n^*} = U ] < E_{t_{n^*}} [ x | s_{n^*} = F ] \), now \( n^* - 1 \) satisfies the initial hypothesis, hence it can replace \( n^* \), and the argument follows by induction.

Next, the second implication follows directly from part 1, and formula (2).

Proof of Theorem 2. Let \( B_{n}^{U} \) and \( B_{n}^{F} \) be the conditional borrowing capacities at time \( t_{n} \). Move back one period, and let \( D \) be the face value of the debt issued at \( t_{n-1} \). Again, to avoid default, \( D \leq \max \{ B_{n}^{U}, B_{n}^{F} \} \).

To simplify notation, for any \( n < N \), let us denote by \( B_{n}^{\max} = \max \{ B_{n}^{U}, B_{n}^{F} \} \), and by \( B_{n}^{\min} = \min \{ B_{n}^{U}, B_{n}^{F} \} \).

If \( s_{n-1} = F \), evaluation of the debt is as usual:

\[
E_{t_{n-1}} [ D | s_{n-1} = F ] = \begin{cases} 
[(1 - q) B_{n}^{U} + q D] \cdot 1_{F} + (1 - 1_{F}) \times \ldots \\
\ldots \times [q B_{n}^{F} + (1 - q) D] \\
B_{n}^{\min} < D \leq B_{n}^{\max} \\
D \quad B_{n}^{\min} \leq D
\end{cases}
\]

where

\[
1_{F} = \begin{cases} 
1 & B_{n}^{\max} = B_{n}^{F} \\
0 & \text{otherwise}
\end{cases}
\]

(5)
while, if \( s_{n-1} = U \), it is:

\[
E_{t_{n-1}} \left[ D \mid s_{n-1} = U \right] = \begin{cases} 
(1 - v) B_n^{\text{min}} + v D & B_n^{\text{min}} < D \leq B_n^{\text{max}} \\
D & B_n^{\text{min}} \leq D
\end{cases}
\]  

(6)

By definition, \( B_{n-1}^* = \max_{D \leq B_n^F} E_{t_{n-1}} \left[ D \mid s_{n-1} = \cdot \right] \), hence

\[
B_{n-1}^F = \max \left\{ B_n^{\text{min}}, (1 - q) B_n^U + q B_n^F \right\} \\
B_{n-1}^U = \max \left\{ B_n^{\text{min}}, (1 - v) B_n^{\text{min}} + v B_n^{\text{max}} \right\}
\]

In the main text, we have shown that \( B_{N-1}^F > B_{N-1}^U \). Hence, by Lemma 1, \( B_{N-2}^F > B_{N-2}^U \). Using (5), (6), and repeated applications of Lemma 1, the inequality can be extended to the entire path, hence \( B_n^F > B_n^U \), \( \forall n \), so that:

\[
B_{n-1}^F = \max \left\{ B_n^U, (1 - q) B_n^U + q B_n^F \right\} \\
B_{n-1}^U = \max \left\{ B_n^U, (1 - v) B_n^U + v B_n^F \right\}
\]

as claimed.

**Proof of Proposition 3.** Let us start with property 1).

As discussed in the Proof of Theorem 2, in the main text we have shown that \( B_{N-1}^F > B_{N-1}^U \). Hence, by Lemma 1, \( B_{N-2}^F > B_{N-2}^U \). Using (5), (6), and repeated applications of Lemma 1, the inequality can be extended to the entire path, hence \( B_n^F > B_n^U \), \( \forall n \).

The second property follows directly from the first one, and formula (3).

The inequality \( B_{n-1}^U \geq B_n^U \) follows from property 1, using (4), and Lemma 1. Finally, suppose there exists \( n^* \), such that \( B_{n^*-1}^U = B_{n^*}^U \), by (4):

\[
B_{n^*-2}^U = \max \left\{ B_{n^*-1}^U, (1 - v) B_{n^*-1}^U + v B_{n^*-1}^F \right\} = \ldots \\
\ldots = \max \left\{ B_{n^*}^U, (1 - v) B_{n^*}^U + v B_{n^*}^F \right\}
\]

By property 2),

\[
\max \left\{ B_{n^*}^U, (1 - v) B_{n^*}^U + v B_{n^*}^F \right\} \leq \ldots \\
\ldots \leq \max \left\{ B_{n^*}^U, (1 - v) B_{n^*}^U + v B_{n^*}^F \right\} = B_{n^*-1}^U = B_{n^*}^U.
\]

Repeated applications of the same argument yield \( B_n^U = B_{n^*}^U \), for any
n ≤ n*. ■

Proof of Proposition 4. As it is clear, \( B_{N}^{F_0} = B_{N}^{F_i} \) and \( B_{N}^{U_0} = B_{N}^{U_i} \).

\[
|B_{n}^{F_i} - B_{n}^{U_i}| = (q - i) |B_{n+1}^{F_i} - B_{n+1}^{U_i}|
\]

\[
|B_{N-1}^{F_0} - B_{N-1}^{U_0}| > |B_{N-1}^{F_0} - B_{N-1}^{U_0}|, \quad \text{since} \quad (q - v) > (q - \tilde{v}).
\]

Hence \( |B_{N-2}^{F_0} - B_{N-2}^{U_0}| > |B_{N-2}^{F_0} - B_{N-2}^{U_0}|. \) This, together with \( (q - v) > (q - \tilde{v}) \), implies:

\[
|B_{N-3}^{F_0} - B_{N-3}^{U_0}| > |B_{N-3}^{F_0} - B_{N-3}^{U_0}|. \quad \text{Repeating this argument for all} \ n
\]

leads to the first property.

Since \( |B_{n-1}^{F_i} - B_{n}^{F_i}| = (1 - q) |B_{n}^{U_i} - B_{n}^{F_i}| \), the second property follows directly from the first one. ■

Proof of Theorem 5. This is precisely Proposition 7 in [1], if we replace \( N + 1 \) by \( N \) and \( V \) by \( \tilde{V} \). ■

Proof of Theorem 6. Starting from the definition of \( B_{1}^{U}(\tau) \), and using Property 3 of Proposition 3, we get:

\[
B_{1}^{U}(\tau) = e^{-\alpha \tau} B_{1}^{U}(\tau) + (1 - e^{-\alpha \tau}) B_{2}^{F}(\tau) \leq ...
\]

\[
\vdots \leq e^{-\alpha \tau} B_{1}^{U}(\tau) + (1 - e^{-\alpha \tau}) B_{2}^{F}(\tau)
\]

For any \( \gamma \), such that \( \gamma < \alpha \), the following inequality holds:

\[
e^{-\alpha \tau} B_{1}^{U}(\tau) + (1 - e^{-\alpha \tau}) B_{2}^{F}(\tau) \leq e^{-\gamma \tau} B_{1}^{U}(\tau) + (1 - e^{-\alpha \tau}) B_{2}^{F}(\tau).
\]

Hence, \( B_{1}^{U}(\tau) \leq e^{-\gamma \tau} B_{1}^{U}(\tau) + (1 - e^{-\alpha \tau}) B_{2}^{F}(\tau) \) for \( \gamma < \alpha \).

Finally, \( B_{1}^{U}(\tau) \leq \left(\frac{1 - e^{-\alpha \tau}}{1 - e^{-\gamma \tau}}\right) B_{2}^{F}(\tau) \).

\( B_{2}^{F}(\tau) \) is bounded by Theorem 5. As \( \tau \to 0 \), both \( e^{-\alpha \tau} \) and \( e^{-\gamma \tau} \) approach to 1. Nevertheless, convergence to zero of the numerator is faster than the one of the numerator since \( \alpha > \gamma \).

Hence \( B_{1}^{U}(\tau) \to 0. \) ■

Proof of Theorem 7. The results for \( N \) and \( N - 1 \) are proved in the main text. For the general case we simply use the proof of Theorem 2, with the generalization that we do not necessarily have \( B_{n}^{F} \geq B_{n}^{U} \). Such a generalization is reflected into the term \( B_{n} \min = \min \{ B_{n}^{U}, B_{n}^{F} \} \), and into the implicit expression for the CEU-evaluation. ■

Proof of Lemma 9. Suppose \( \exists n^*, 1 \leq n^* \leq N \), such that \( E_{t_{n^*}} [x | s_{n^*} = U] \leq E_{t_{n^*}} [x | s_{n^*} = F] \), hence:

\[
E_{t_{n^* - 1}} [x | s_{n^* - 1} = F] = qE_{t_{n^*}} [x | s_{n^*} = F] + (1 - q) E_{t_{n^*}} [x | s_{n^*} = U]
\]

and
Given (2), and
\[ E_{t_{n^*-1}} [x|s_{n^*-1} = U] = \ldots \]
\[ \ldots = \min_{(\mu, 1-\mu) \in C(v)} \{ \mu E_{t_{n^*}} [x|s_{n^*} = F] + (1-\mu) E_{t_{n^*}} [x|s_{n^*} = U] \} \]

Given (2), and \( E_{t_{n^*}} [x|s_{n^*} = U] \leq E_{t_{n^*}} [x|s_{n^*} = F] \)
\[ E_{t_{n^*-1}} [x|s_{n^*-1} = U] = v(F) E_{t_{n^*}} [x|s_{n^*} = F] + \ldots \]
\[ \ldots + (1 - v(F)) E_{t_{n^*}} [x|s_{n^*} = U] \]

Since \( v(F) > q \)
\[ E_{t_{n^*-1}} [s_{n^*-1} = U] > E_{t_{n^*-1}} [s_{n^*-1} = F] \]

Next consider \( n^* - 2 \).
\[ E_{t_{n^*-2}} [x|s_{n^*-2} = F] = q E_{t_{n^*-1}} [x|s_{n^*-1} = F] + (1 - q) E_{t_{n^*-1}} [x|s_{n^*-1} = U] \]
and
\[ E_{t_{n^*-2}} [x|s_{n^*-1} = U] = \ldots \]
\[ \ldots = \min_{(\mu, 1-\mu) \in C(v)} \{ \mu E_{t_{n^*-1}} [x|s_{n^*} = F] + (1-\mu) E_{t_{n^*-1}} [x|s_{n^*} = U] \} \]

Given (2), and \( E_{t_{n^*-1}} [x|s_{n^*-1} = U] > E_{t_{n^*-1}} [x|s_{n^*-1} = F] \)
\[ E_{t_{n^*-2}} [x|s_{n^*-1} = U] = (1 - v(U)) E_{t_{n^*}} [x|s_{n^*} = F] + \ldots \]
\[ \ldots + v(U) E_{t_{n^*}} [x|s_{n^*} = U] \]
\[ (1 - v(U)) \geq v(F) > q \]

Further, since \( E_{t_{n^*-1}} [x|s_{n^*-1} = U] > E_{t_{n^*-1}} [x|s_{n^*-1} = F] \), we get
\[ E_{t_{n^*-2}} [x|s_{n^*-2} = U] < E_{t_{n^*-2}} [x|s_{n^*-2} = F] \]

Replacing \( E_{t_{n^*}} [x|s_{n^*} = U] \) and \( E_{t_{n^*}} [x|s_{n^*} = F] \) by \( E_{t_{n^*-2}} [x|s_{n^*-2} = U] \) and \( E_{t_{n^*-2}} [x|s_{n^*-2} = F] \), respectively, the same argument as before applies, which provides the desired result.
Appendix Figures

Figure 1: Nonfinancial, Financial and ABCP markets, outstanding values in billions of dollars. (Bloomberg)
Figure 2: Aggregate Bank Credit Default Swap Rate and Selected Spreads (In basis points). Notice that whereas during the pre-crisis period the processes are approximately constant, a structural break in their respective data generating processes is evident in the Summer 2007.
Figure 3: Bear Stearns’ capital and liquidity positions. Source Chairman Cox Letter to Basel Committee in Support of New Guidance on Liquidity Management.
References


