Labor Market Participation, Unemployment and Monetary Policy∗

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Abstract

In the present paper we study to what extent the introduction of endogenous participation in an otherwise standard DSGE model with matching frictions plays a role for business cycle dynamics and monetary policy.

Keyword: matching frictions, endogenous participation, monetary policy.

JEL Codes: E24, E32, E52

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1 Introduction

Despite the rate at which the macro literature has produced papers studying the interaction between labor market frictions and the aggregate economy\(^1\) little to no attention as been devoted to the choice of workers to participate in the labor market. The question we address in this paper is to what extent the participation margin plays a role for: business cycle dynamics; transmission mechanism of monetary policy; monetary policy design.

In models with matching frictions without participation margin, households cannot decide to enter and leave the labor market in response to aggregate fluctuations. As an implication, involuntary unemployment can never be substituted with voluntary non-employment. Also, households’ decisions have no impact on labor market tightness, which is entirely determined by firms. We ask whether the participation margin allows households to circumvent labor market frictions or at least to improve upon a situation where labor supply is perfectly inelastic. The paper shows that the answer depends on households’ incentives to vary the participation margin. In turn, those incentives are driven by the cost of search, the job finding probability and the marginal rate of substitution between market and home production activity. Importantly, the way they interact is not independent of nominal price rigidities.

Since we want to be able to easily compare our model with a standard frictionless model and with a model with matching frictions but exogenous participation, we introduce endogenous participation in the simplest possible way. At the beginning of each period agents can be employed, unemployed and actively searching for a job or non-participant. Non participant agents allocate all their time to home production activity\(^2\). Employed agent allocate all their time to market activity and receive a wage in exchange. Searching workers allocate a certain fraction of their time to actively searching for a new job. If matched with a firm they start working immediately otherwise they will allocate the remaining time to home production. Following the tradition in this literature we assume that agents belongs to big families where they pull together home production activity, wage and unemployment benefit and share the same consumption so that we do not need an heterogeneous agents model. Other than this, the model is completely standard featuring matching frictions à la Mortensen and Pissarides (1999) and nominal price rigidity à la Calvo (1983).

The main results of the paper can be summarized as follow. First, the household always aims at replicating the level of home production in the frictionless


\(^2\)You can think that each agent is endowed with 24h out of which 8 are allocate to sleep and 8 to leisure. The allocation of the last 8 depends on the participation decision and the matching frictions.
model, under all parametrization, be prices sticky or not. Then, the main incentive driving participation decisions is to keep the marginal rate of substitution between market and non-market consumption at the level that would prevail absent labor market frictions. Second, it is evident that the participation decision always interacts with nominal price rigidity. In fact, under sticky prices firms are demand constrained, this leading to undesired fluctuations in mark-ups and as a consequence in the marginal rate of substitution. However, those fluctuations cannot be undone by the participation decision. It follows that price stickiness impacts on the participation decision through the ideal allocation the household tries to replicate. Third, movements in the job finding rate are also an important determinant of the participation decision. Forth, the conduct of monetary policy influences the way technology shocks are transmitted to the economy and this is true for all the models. Fifth, the way monetary policy interacts with the transmission mechanism of a technology shock in a model with matching frictions depends on whether there is exogenous or endogenous participation. For example, while switching from a frictionless model to one with matching frictions and exogenous participation has only a level effect on GDP while preserving the same sensitivity of the variable to a change in monetary policy, this is not true when allowing for endogenous participation together with matching frictions. In particular, moving from a coefficient of 1.5 to a coefficient of 10 makes GDP increasing by an extra 9.3% while for the other two models the extra increase is only 4.5%. Finally, the cost of search activity (measured in terms of time allocated to it and thus diverted from home production) plays a crucial role in shaping the transmission mechanism. More precisely, when the cost is high that the household decreases participation to undo the effects of price rigidity. In such context a more aggressive monetary policy, by making the world closer to flexible prices, would induce a smaller reduction in participation after a technology shock than a loose monetary authority. If instead we leave in a world where the search cost is low, the household wants to reduce participation after a technology shock in order to offset the increase in the finding rate. And the more so the more prices are flexible. Thus in such a world a more aggressive monetary policy would achieve a bigger reduction in participation, exactly the opposite of before. Therefore one might think that in the first case there is little or no trade-off faced by the monetary policy i.e. there is no bias against price stability while this might not be the case if the cost of search activity is low. A formal welfare analysis is needed to verify whether this intuition is indeed correct.

Though endogenous participation has been overlooked from the literature up until now, we are not the first studying it. Garibaldi and Wasmer (2005) introduced participation in a static framework where agents differ in the value of their home productivity. Ebell (2008) modelled the participation margin in a real business cycle model. We follow her in adopting a modelling strategy that eliminates agents heterogeneity but we do allow unemployed workers to take part in the home production activity. This is a crucial difference since we show in the paper that assuming an extremely high cost of search has important implications for both the transmission mechanism in a real business cycle framework,
and for monetary policy once price rigidity is introduced. Haefke and Reiter (2006) solve an heterogeneous agents model in a real business cycle framework. Recently, Bruckner and Pappa (2010) introduced endogenous participation in a New Keynesian model to study the transmission of fiscal shocks on labor market variables. Like Ebell (2008), they also assume no home production from unemployed workers. Gali (2010) focuses on the interaction between matching frictions and optimal monetary policy in a New Keynesian model where the cost of search is high. Finally, Christiano, Trabandt and Walentin (2010) introduced the participation margin in a medium-scale New Keynesian model with household heterogeneity where however the finding rates are exogenously assumed instead of arising from the presence of matching frictions.

2 The Model

The representative household consists of a continuum $[0, 1]$ of family members. Each of them can be employed, unemployed or non participant. Non participant family members allocate all their time to home production. Employed members spend all their time to work receiving a salary in exchange. Unemployed workers spend some of their time actively searching for a new job while the rest is used for home production. While unemployed, they are entitled to an unemployment benefit. Wages, unemployment benefits and home production are pulled together and redistributed equally within the family members so that they all enjoy the same level of consumption and home production\(^3\). Consumption and savings are decided at the household level, together with the choice of how many family members to let participate in the labor market.

The economy is characterized by two sectors\(^4\). In the final sector there is a continuum of retailers, each selling a differentiated good under monopolistic competition and using intermediated goods as the only input in production. Calvo price stickiness is assumed in this sector. In the intermediate sector infinitely many firms produce an homogeneous good under perfect competition and flexible prices. In order to produce each firms has to be matched with a worker. Firms are subject to a vacancy posting cost when searching for a worker. Existing matches can be exogenously discontinued at any time.

2.1 Households

A household is made up by a continuum $[0, 1]$ of family members. Let $E_{t-1}$ be the employed members in period $t - 1$. When entering period $t$, a fraction $\rho$ of those jobs will be exogenously discontinued. Among those, some may drop out of the labor force, if the household decides to reduce labor market participation, 

\(^3\)See Andolfatto (1996) and Merz (1995).

\(^4\)We use the two sectors set-up in order to keep the matching frictions separated from the price rigidity. See for example Sveen and Weinke (2008).
while the others will search for a new job. We assume instantaneous hiring\(^5\) i.e. searching workers matched with a firm will start working already in period \(t\). Searching workers who will not be matched, will receive the unemployment status, be entitled to the unemployment benefit and take part to some home production\(^6\). Therefore, if \(N_t\) is the fraction of family members participating in the labor market, searching workers in period \(t\) are defined as\(^7\):

\[
S_t = N_t - (1 - \rho)E_{t-1}
\]

while non participant members are given by:

\[
L_t = 1 - N_t
\]

Let \(f_t\) be the job finding rate, that will be endogenously defined when solving the search and matching problem in the intermediate sector. Then, the evolution of employment reads as follow:

\[
E_t = (1 - \rho)(1 - f_t)E_{t-1} + f_tN_t
\]

Let \(C_t \equiv \left[\int_0^1 C_t(i)^{\frac{1-\varepsilon}{\varepsilon}} \, di\right]^\frac{\varepsilon}{1-\varepsilon}\) be a Dixit-Stiglitz aggregator of different varieties of goods. The optimal allocation of expenditure on each variety is given by \(C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t\) where \(P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} \, di\right]^\frac{1}{1-\varepsilon}\). The representative household then chooses aggregate consumption, savings and participation in order to maximize the expected lifetime utility\(^8\):

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) + \phi \frac{(1 - E_t - \Gamma U_t)^{1+\nu}}{1 + \nu} \right]
\]

subject to:

\[
P_t C_t + R_t^{-1} D_t \leq D_{t-1} + W_t E_t + P_t b U_t + T_t
\]

\[
E_t = (1 - \rho)(1 - f_t)E_{t-1} + f_tN_t
\]

\(^{5}\)Because of the assumption of sticky prices, production is demand driven in the short run. Therefore, firms need to have a margin of adjustment to supply as many goods as demanded at the prevailing price. In a model without capital, as standard in this literature, there are two options to achieve this. Either introduce endogenous job destruction or allow for instantaneous hiring. We decided for the second in order to keep the model as simple as possible. Since we calibrate the model at quarterly frequency, it also seems reasonable.

\(^{6}\)Intuitively, we are assuming that the search process takes place at the beginning of the period so that, if matched with a vacancy, workers can produce immediately otherwise, they can use the rest of their time for home production.

\(^{7}\)We are implicitly assuming that when reducing participation, there are always enough unemployed workers out of which to choose so that all workers who were employed in the previous period and whose job was not exogenously discontinued, will keep their job.

\(^{8}\)We model endogenous participation in a way similar to Ebell (2008) though we do allow unemployed workers to allocate some of their time to home production.
\[ N_t = E_t + U_t \]  

where \( W_t \) is the nominal wage, \( b \) is the real unemployment benefit, \( D_t \) is a one period head nominal bond, \( T_t \) contains lump-sum taxes and profits, \( h_t \equiv 1 - E_t - \Gamma U_t \) represents the home production activity, \( 0 < \Gamma < 1 \) is the fraction of time that unemployed workers devoted to the search activity and \( \nu < 0 \) is the inverse of the home production elasticity.

The first order condition with respect to the home production \( h_t \) is:

\[
\left[ 1 - \frac{f_t}{f_t} \right] (\phi \Gamma h_t \nu C_t - b) = \frac{W_t}{P_t} - \phi h_t \nu C_t + \beta E_t \left\{ \frac{C_t}{C_{t+1}} \left[ 1 - \rho \right] \left( 1 - f_{t+1} \right) \left( \phi \Gamma h_{t+1} \nu C_{t+1} - b \right) \right\}
\]

while the optimal intertemporal condition is the usual Euler equation:

\[
\beta R_t E_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} = 1
\]

### 2.2 Firms

#### 2.2.1 Intermediate Good Producers

There are infinitely many firms \( j \in [0, 1] \) producing an homogeneous good under perfect competition and flexible prices and using labor as the only input in production. The labor market is characterised by matching frictions in the standard Mortensen and Pissarides (1999) framework. Firms have to search for a worker in the pool of searching workers. Posting a vacancy costs \( \kappa \) units of the final good \( C_t \) in each period. When the vacancy is filled, it produces:

\[ X_t(j) = A_t \]

where the (log of) technology \( A_t \) is assumed to follow an AR(1) process: \( \log(A_t) = \rho \log(A_{t-1}) + \xi_t^a \) with \( \xi_t^a \) being an i.i.d. shock with zero mean and variance \( \sigma_a \).

We use a standard constant return to scale technology converting searching workers \( S_t \) and vacancies \( V_t \) into new matches \( M_t \):

\[ M_t = \omega V_t^{1-\gamma} S_t^\gamma \]

We define labor market tightness as \( \theta_t \equiv \frac{V_t}{S_t} \), the job filling rate (i.e. the rate at which searching workers meet a vacancy) as \( q_t \equiv \omega \theta_t^{-\gamma} \), and the job finding rate (i.e. the rate at which vacancies are filled) as \( f_t \equiv \theta_t q_t \). Because of instantaneous hiring, once the vacancy is filled it is immediately productive. Let \( P_t \) be the price at which firms sell the homogenous good to the final goods producers. The value of a filled vacancy, \( V_t^f \) expressed in terms of the final consumption bundle \( P_t \), is given by:
\[
V_t^J = \frac{P_t^x}{P_t} A_t - \frac{W_t}{P_t} + (1 - \rho)E_t \left\{ Q_{t,t+1} V_{t+1}^J \right\}
\] (2.12)

where \( Q_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}} \). The free entry condition ensures that:
\[
\frac{\kappa}{q_t} = V_t^J
\] (2.13)

Substituting (2.13) into (2.12) gives the job creation condition:
\[
\frac{\kappa}{q_t} = \frac{P_t^x}{P_t} A_t - \frac{W_t}{P_t} + (1 - \rho)E_t \left\{ Q_{t,t+1} \frac{\kappa}{q_{t+1}} \right\}
\] (2.14)

Finally, the wage is determined solving a Nash bargaining problem between the firm and the worker. In order to do that we have to compute the surplus from employment keeping participation constant. This is given by:
\[
V_t^w = W_t \frac{P_t}{P_t^x} - b - \phi h_t^x (1 - \Gamma) C_t + E_t \left\{ Q_{t,t+1} (1 - \rho) \left[ 1 - f_{t+1} \right] V_{t+1}^w \right\}
\] (2.15)

Let \( \eta \) be the firm’s bargaining power. Then, the total surplus form the match is split according to the optimal sharing rule:
\[
\eta V_t^w = (1 - \eta) V_t^J
\] (2.16)

Using the definitions of \( V_t^J \) and \( V_t^w \) in (2.16), together with the free entry (2.13) and the job creation condition (2.14), it is possible to derive the wage equation:
\[
\frac{W_t}{P_t} = (1 - \eta) \frac{P_t^x}{P_t} A_t + \eta \left[ b + \phi h_t^x (1 - \Gamma) C_t \right] + (1 - \eta) \left[ 1 - \rho \right] E_t \left\{ Q_{t,t+1} \kappa \theta_{t+1} \right\}
\] (2.17)

### 2.2.2 Final Goods Retailers

In the final good sector there are infinitely many producers of differentiated goods. Each is producing a variety \( i \in [0, 1] \) using the following technology:
\[
Y_t(i) = X_t(i)^{1-\alpha}
\] (2.18)

They face a downward sloping demand function\(^{10}\):
\[
Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\gamma} \left[ C_t + \kappa V_t \right]
\] (2.19)

Under flexible prices the optimal pricing rule is given by:

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9 See appendix for the derivation.
10 Remember that intermediate firms pay the vacancy posting cost in terms of final goods and therefore solve an expenditure minimization problem like the household.
where $P_t^*(i)$ is the optimal price and $MC_t(i) = \frac{1}{1-\alpha} P_t X_t(i)^\alpha$ is the nominal marginal cost. Imposing symmetry equation (2.20) becomes:

\[
1 = \frac{\varepsilon}{\varepsilon - 1} \frac{1}{P_t} \frac{P_t^x}{P_t} X_t^\alpha
\]  

When price rigidity à la Calvo (1983) is assumed, the pricing first order condition for a firm allowed to reoptimize in $t$ is given by:

\[
\sum_{T=0}^{\infty} \xi^T E_t \left\{ Q_{t,t+1} Y_{t+T}(i) \left[ P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} MC_t(i) \right] \right\}
\]

where $\xi$ represents the probability of not changing the price in a given period.

### 2.3 Market Clearing Conditions

The aggregate production of the intermediate sector is given by:

\[
X_t = \int_0^1 X_t(j) dj = A_t E_t
\]

Integrating the demand of good $i$, (2.19) yields the conventional aggregate resource constraint:

\[
Y_t = C_t + \kappa V_t
\]

after defining aggregate output as:

\[
Y_t = \left\{ \int_0^1 Y_t(i)^{\frac{\varepsilon}{\varepsilon - 1}} di \right\}^{\frac{\varepsilon}{\varepsilon - 1}}
\]

Combining demand of final goods (2.19) with their production function and integrating delivers the aggregate production function:

\[
Y_t = X_t^{1-\alpha} \Delta_t^{\alpha-1}
\]

where the following definition applies:

\[
\Delta_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{\frac{-\varepsilon}{\alpha}} di
\]

and $\Delta_t$, bounded by 1 from below, is a measure of price dispersion.
2.4 Equilibrium

Log-linearizing (2.22) around the zero inflation symmetric steady state we obtain the New Keynesian Phillips Curve (NKPC):

\[ \hat{\pi}_t = \beta \hat{\pi}_{t+1} + \lambda \left( \frac{\hat{P}_x}{\hat{P}_t} + \alpha \hat{x}_t \right) \] (2.28)

where \( \lambda = \frac{(1-\xi)(1-\beta \xi)}{\xi} \frac{1-\alpha}{1-\alpha+\alpha \xi} \) and variables with a hat represent log-deviations from steady state.

2.5 Monetary Policy

We assume that the monetary policy follows a simple interest rate rule:

\[ \log(R_t) = \log(\beta) + \phi \hat{\pi}_t \] (2.29)

3 Endogenous Participation

Rearranging the optimality condition (2.8) allows to gain some insight about the key determinants of the participation decision. After defining:

\[ \Omega_t \equiv (1 - f_t) f_t \left[ \phi \Gamma h_t C_t - b \right] \] (3.1)

(2.8) can be rewritten recursively as:

\[ \Omega_t = \frac{W_t}{P_t} - \phi h_t C_t + E_t \left( \frac{\beta C_t (1-\rho)}{C_{t+1}} \Omega_{t+1} \right) \] (3.2)

Note that \( \phi \Gamma h_t C_t - b \) is the flow benefit of withdrawing one unemployed worker from the labor force and reallocating it to home production in terms of consumption, net of the unemployment benefit. Also, the term \( \left[ \frac{1-f_t}{f_t} \right] \) is a wedge introduced by matching frictions capturing the extra change in home production, relative to a frictionless labor market, needed to increase employment by one unit. In fact, by manipulating the law of motion of employment, it is straightforward to get:

\[ E_t = (1-\rho)E_{t-1} + \frac{f_t}{1-f_t} U_t \] (3.3)

Not surprisingly, matching frictions introduce a wedge between employment and the participation decision. Such a wedge decreases in the job finding rate and it is strictly positive for a job finding rate lower than one.

We interpret (3.2) as the optimality condition for labor market participation: it states that the marginal benefit of increasing employment has to equalize its
marginal cost, once the wedge due to frictions is taken into account. On the one hand, $\Omega_t$ is the utility loss implied by diverting from home production the extra fraction of population frictions require to marginally increase employment. On the other, the right hand side of (3.2) represents the household’s marginal benefit, adding the wage premium over the marginal rate of substitution to the option value of getting an additional member into employment, $\Omega_{t+1}$. A positive option value arises, as long as a match realized in the current period allows the household to save on the future search cost with a positive probability $1 - \rho$. Finally, note that if the wedge vanishes, the marginal rate of substitution between consumption and home production equals the real wage. We define such a situation as full participation, since non-employment is entirely voluntary.

Equation (3.2) reveals the main aggregate forces driving participation: the job finding rate, the marginal rate of substitution and the opportunity cost of searching a job rather than staying at home, $\Gamma$.

A raise in the finding rate shifts downwards the marginal cost of increasing employment, for any given level of the marginal rate of substitution. Therefore, everything else equal, home production has to fall as leisure would do in the baseline business cycle model with endogenous labor supply. The effect of finding rates is magnified by the cost of search. In fact, $\Gamma$ captures the additional drop in home activity the household has to suffer from sending members to the market whenever there are matching frictions. The higher is the cost of search, the more profitable is to reallocate members between market and non-market activity, following a change in the finding rate. In addition, the opportunity cost of search shapes the equilibrium relation between home production and participation. This becomes evident by looking at the following equation

$$h_t = 1 - E_t - \Gamma U_t = 1 - N_t + (1 - \Gamma)U_t$$

(3.4)

focusing on two extreme cases: $\Gamma = 1$ and $\Gamma = 0$. $\Gamma = 1$ means that unemployed workers use up all their time in search activity and thus do not contribute to home production, exactly like employed workers. Therefore home production moves one to one with participation while employment and unemployment mirror image each other. When $\Gamma = 0$, search activity is costless in terms of home production and employment match movements in $h$. The intuition is that being a member of the labor force is costly only to the extent that a match is realized, when the cost of search is low. In contrast, when it is high, it is participation to be costly, irrespectively of how the labor force is split in employment versus unemployment. For the interpretation of our results, it is important to keep in mind that equation (3.4) holds irrespectively of whether participation is endogenous or not. The optimality condition for participation determines endogenously the desired level of home production. Then, technology shapes the relation between home production and labor market variables. However, when participation is exogenous and constant, $h$ cannot be chosen and it is determined by technology itself. Hence, when $\Gamma$ is high, home production needs to be constant, as well as participation. When $\Gamma$ is low, home production is determined by frictions, since
\[ E_t = (1 - \rho)(1 - f_t)E_{t-1} + f_t N_t \]  

and the finding rates fluctuates exogenously to the household.

Finally, it is also important to recall for the analysis below that the substitution effect between home and market activity can be overturned by a wealth effect. Indeed, a sufficiently large surge in consumption associated to increasing finding rates can push home production into the opposite direction.

4 Participation, Matching Frictions and Sticky Prices

4.1 Parametrization and Steady State

We choose most of parameters as it is conventional in the literature, as we briefly summarize below. Particular emphasis is placed on those parameters that are crucial in shaping the effects of endogenous participation.

First, we restrict to the case of a deterministic steady state where inflation and productivity are constant and normalized to zero and one respectively. It follows that the relative price dispersion of the final goods is zero, while the relative price of the intermediate good is distorted only by monopolistic competition in the final good sector.

As it is standard in the New-Keynesian literature, the elasticity of substitution among varieties of the final good is set to 6 and the Calvo parameter is \( \xi = \frac{2}{3} \). The law of motion of employment (2.3) gives a relation between the steady state employment rate, the finding rate and the exogenous separation rate

\[ \frac{E}{N} = \frac{f}{1 - (1 - \rho)(1 - f)} \]  

We set the separation rate, \( \rho \), to 0.1 following Shimer (2005) and by targeting an employment rate of 0.9520 we recover the implied finding rate, 0.70 per quarter, which is lower than in Shimer (2005). A lower finding rate is explained by the assumption of instantaneous hiring. In fact, workers can be matched in the same period they start to search, so that the model needs a lower \( f \) to replicate the same employment rate. The scaling parameter of the matching function, \( \omega \), is chosen in such a way that the job filling rate \( q \) is equal to 2/3. This implies a steady state labor market tightness of about 1. These values are conventionally used in the literature, though it is worth noticing that all our results are robust to changes of the steady state of \( q \) and \( \theta \). In fact, as pointed out by Shimer (2005), the value of those variables is simply a matter of normalization. Finally, we restrict to 0.5 the elasticity of matches to vacancies.

Then, we are left with six free parameters: the inverse of labor supply elasticity \( \nu \), the preference shifter \( \phi \), the cost of search \( \Gamma \), workers’ bargaining power \( 1 - \eta \), the unemployment benefit \( b \) and the cost of posting a vacancy \( \kappa \). We
set $\nu = -5$ in order to keep comparability with Gali (2010). It is important to keep in mind that this choice for the inverse of the labor supply elasticity generates a too volatile participation if workers’ bargaining power is set to a standard $1 - \eta = 0.5$. The solution adopted by Gali (2010) is to use $1 - \eta = 0.95$ instead. In this section, we leave free the bargaining power. Therefore, we will perform all the exercises we are interested in for different values of $\eta$ and we will show that the role of fluctuations in the labor force may be substantially different, depending on workers’ bargaining power. The explanation of the underlying economic intuition is among the objects of the following section. Unemployment benefit is determined so as to target a replacement rate of 20 percent at the steady state. As far as the choice of other parameters is concerned, we follow closely Hagedorn and Manovskii. In particular, we use the job creation equation to determine the real wage and $\kappa$ by matching the cost of posting a vacancy per filled job as a fraction of the real wage. We choose 4.5 percent as a target. Then, $\Gamma$ is pinned down by the wage equation for given bargaining power, after replacing the marginal rate of substitution with its steady state value

$$\phi h^\nu c = \frac{fW/P + (1 - f)(1 - \beta(1 - \rho))b}{\Gamma(1 - f)(1 - \beta(1 - \rho)) + f}$$ (4.2)

Finally, we can determine ex-post the value of $\phi$ implementing the observed participation rate $N_t = 0.69$.

### 4.2 Impulse Responses

To disentangle the different forces behind the participation decision, we compare the impulse responses to a positive one percent TFP shock for three versions of our model: a version with Walrasian labor markets and variable home production, to which we refer as the frictionless labor market case (or frictionless for short); a version with matching frictions but exogenous labor market participation; our full model with both matching frictions and endogenous labor market participation. Note that all versions feature the same steady state, apart from unemployment, which is constantly equal to zero, and vacancies and labor market tightness, which do not appear in the frictionless labor market model. Note also that, conditionally on prices being flexible, the assumption of log utility in consumption implies that home production in the frictionless model does not respond to TFP shocks. Its steady state value instead coincides across the three versions. As an implication, whenever home production reacts to shocks, it does so to respond to changes in the finding rates. The baseline comparison is done assuming that workers’ bargaining power is $1 - \eta = 0.95$. We study both the case of flexible (Figure 1) as well as sticky prices (Figure 2). We then analyze how results would change under the more traditional assumption of $1 - \eta = 0.5$ (Figures 3 and 4). Before looking at the impulse response functions, it is instructive to recall that the way the model is parameterized creates a link between $\eta$ and the cost of search $\Gamma$. In addition, changes in the cost of search yields a different steady state opportunity cost of being employed, relatively to be unem-
Table 1: Relation between $1 - \eta$, $\Gamma$ and the value of leisure

<table>
<thead>
<tr>
<th></th>
<th>0.95</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
<td>0.8291</td>
<td>0.2321</td>
</tr>
<tr>
<td>Value of leisure</td>
<td>0.3654</td>
<td>0.9666</td>
</tr>
</tbody>
</table>

ployed, obviously conditionally on choosing to participate to the labor market. We express this opportunity cost in consumption equivalents

\[
\frac{(1 - \Gamma)\phi h'^{\nu}C}{W/P}
\]

and, as it is common in the matching friction literature, we refer to it as to the value of leisure. $\Gamma$ and the value of leisure implied by different parametrizations of $1 - \eta$ are reported in Table 1.

Figure 1 displays the case of flexible prices and high bargaining power of workers, which is in turn implying a high cost of search. When the labor market is frictionless and prices are perfectly flexible, involuntary unemployment is constantly equal to zero, while employment and participation coincide and they remain constant at their steady state level. Therefore, absent any friction, home production is constant. When matching frictions are introduced, employment and unemployment fluctuate in response to the shock and they display the same dynamics irrespectively of whether the participation margin is active or not. In fact, the change in participation is modest and only slightly affects home production. Hence, the participation margin does not seem to matter. The intuition goes as follows. When $\Gamma$ is high, $h_t$ tends to $1 - N_t$. Therefore, if $N$ is exogenous and constant, home production tends to be constant too. This makes the desired home production level defined by the frictionless model and the actual one under exogenous participation relatively close to each other. Hence, participation does not move even if it can, and ultimately does not matter for labor market variables.

In Figure 2, sticky prices are introduced keeping the same parametrization as in Figure 1. Again, the household chooses participation so as to replicate the equilibrium that would prevail with flexible labor markets in terms of home production. Because of stickiness, the desired level of non-market activity positively responds due to a contraction in employment in the frictionless model. In contrast, under exogenous participation and matching frictions home production does not fluctuate since, like in the previous case, $h_t$ tends to $1 - N_t$, which is constant by definition. It follows that matching frictions are binding and prevent the household from optimally solving the employment-home production trade-off: unemployment, which is positive at the steady state because of frictions, does not fall as consumers would like to in order to increase non-market activity. Hence, the household reduces participation when she is allowed to do so, leading to a fall in employment and unemployment.
Figure 3 repeats the same exercise as Figure 1, but with a lower value of workers’ bargaining power, which implies a low cost of search. When $\Gamma$ is low, $h_t$ tends to $1 - E_t$. As emphasized in the previous section, this implies that home production under exogenous participation is determined by frictions and it has to fall after a positive productivity shock, since the finding rate increases. This opens a gap with respect to the desired level, which instead remains constant. Then, active the participation margin the household withdraws unemployed from the labor force and as a result participation and unemployment fall. Contrary to Figure 1, matching frictions bind and the participation margin matters.

Finally, in Figure 4 nominal rigidities are introduced. Because of stickiness the finding rate falls under exogenous participation, making unemployment and home production increase. The magnitude of the effect is such that non-market activity increases as much as it would in the frictionless case, making matching frictions non binding. This explains why, differently from the previous calibration, the endogenous and the exogenous participation model are closer under sticky prices, rather than under flexible. Also, and again differently from the case of $\eta = 0.05$, participation gets less volatile when prices are sticky.

Therefore, some conclusions can be drawn looking at impulse responses:

- First, Figures 1-4, taken altogether, strikingly display a common pattern in terms of home production. The household always aims at replicating frictionless home goods consumption, under all parametrization, be prices sticky or not. Then, the main incentive driving participation decisions is to keep the marginal rate of substitution between market and non-market consumption at the level that would prevail absent labor market frictions.

- Second, it is evident that the participation decision always interacts with nominal price rigidity. In fact, under sticky prices firms are demand constrained, this leading to undesired fluctuations in mark-ups and as a consequence in the marginal rate of substitution. However, those fluctuations cannot be undone by the participation decision. It follows that price stickiness impacts on the participation decision through the ideal allocation the household tries to replicate.

- Third, movements in the job finding rate are also an important determinant of the participation decision.

- Forth, the way the desired allocation the household tries to replicate transmits to labor market variables depends on the cost of search. For a high cost of search, participation is adjusted independently of frictions so as to implement the desired allocation. For a low cost of search, employment adjusts to home production and participation varies to match unemployment fluctuations, making stronger the substitution of unemployment with voluntary non-employment.

More intuition on the forth point is provided in the next section.
5 Participation and monetary policy: the interaction

Having gained some intuition on the interactions between matching frictions, endogenous participation and sticky prices for given monetary policy, we now study whether and how those interactions depend on monetary policy. More precisely, we ask two questions: can monetary policy influence the transmission mechanism of shocks in a model with matching frictions, sticky prices and endogenous participation? Does it so in a way that is substantially different from the case of exogenous participation?

In the impulse response analysis we set the reaction coefficient of the interest rate rule to $\phi_\pi = 1$. We now study the implications of letting this coefficient vary from 1.5 to 10 (which we consider to be a proxy for strict inflation targeting). Since we want to compare three models across several monetary policy reaction coefficients, and in order to make the comparison as simple as possible, we focus on the reaction of each variable in the first period the shock realises (impact response)$^{11}$. More precisely, Figure (5) shows the impact response of each variable to a technology shock for different values of $\phi_\pi$ while keeping all the other parameters at their baseline calibration. Therefore, the steeper the curve, the higher is the sensibility of a variable to changes in monetary policy. Figure (6) does the same for the case $\eta = 0.5$.

Under the baseline calibration $\eta = 0.05$, we have seen that the cost of search $\Gamma$ is high and there is almost no difference between endogenous and exogenous participation when prices are flexible while this is not the case under sticky prices. Therefore, strict targeting aligns endogenous and exogenous participation in terms of marginal rate of substitution while loose monetary policy makes endogenous and exogenous further away from each other (Figure 5). This implies that it is stickiness, rather than matching frictions alone, that induces households to substitute $U_t$ with $N_t$. Hence, under loose monetary policy households reduce unemployment via a reduction in labor market participation. Under strict inflation targeting instead, we replicate flex prices therefore unemployment decreases by more while participation moves very little. Therefore, the sign of the correlation is always positive conditional on a technology shock but smaller than before. Hence, monetary policy changes not only the volatility of the variables but also the magnitude of their correlation.

Interestingly, we obtain exactly the opposite result under the alternative calibration, $\eta = 0.5$ (Figure 6) where the cost of search $\gamma$ is low. The intuition is pretty simple. When the cost of search is low the models with exogenous and endogenous participation are further away under flex than under sticky prices, the opposite of what was happening under the baseline calibration. Indeed under flexible prices households reduces participation to undo the increase in the

$^{11}$The only exception being the searching workers for the model with exogenous participation for which we report the response one period after the shock. We do so given that for this particular model searching workers are a state and cannot move in the first period.
finding rate that follows the positive technology shock, in order to smooth move-
ments in home production. As a consequence of that, it is now loose monetary
policy that makes endogenous and exogenous close, while strict targeting makes
them further away. Then households tend to substitute \( U_t \) with \( N_t \) under strict
targeting, and as a consequence the conditional correlation of unemployment and
participation is higher in this case.

We have seen that the two calibrations predict different impacts of changes
in the monetary policy on the correlation between unemployment and partici-
pation. But there is also another crucial difference between the two. When \( \Gamma \)
is high it is the interaction between endogenous participation and price rigidity
that makes household substitute \( U_t \) with \( N_t \). When instead \( \Gamma \) is low, it is the in-
teraction between endogenous participation and matching frictions that induces
the substitution.

We can summarize the main results derived up to now as follows. First, the
conduct of monetary policy influences the way technology shocks are transmitted
to the economy and this is true for all the models. Second, the way monetary pol-
icy interacts with the transmission mechanism of a technology shock in a model
with matching frictions depends on whether there is exogenous or endogenous
participation. For example, while switching from a frictionless model to one with
matching frictions and exogenous participation has only a level effect on GDP
while preserving the same sensitivity of the variable to a change in monetary
policy, this is not true when allowing for endogenous participation together with
matching frictions. In particular, moving from a coefficient of 1.5 to a coefficient
of 10 makes GDP increasing by an extra 9.3% while for the other two models
the extra increase is only 4.5%. Third, the cost of search activity \( \Gamma \) plays a
crucial role in shaping the transmission mechanism. More precisely, when the
cost is high we have seen that the household decreases participation to undo
the effects of price rigidity. In such context a more aggressive monetary policy,
by making the world closer to flexible prices, would induce a smaller reduction
in participation after a technology shock than a loose monetary authority. If
instead we leave in a world where the search cost is low, the household wants
to reduce participation after a technology shock in order to offset the increase
in the finding rate. And the more so the more prices are flexible. Thus in such
a world a more aggressive monetary policy would achieve a bigger reduction in
participation, exactly the opposite of before. Therefore one might think that in
the first case there is little or no trade-off faced by the monetary policy i.e. there
is no bias against price stability while this might not be the case if the cost of
search activity is low. However, in order to be sure of this we need to study the
welfare.

6 Conclusions

We introduced endogenous participation in an otherwise standard New Keynes-
sian model with matching frictions. We used this laboratory economy to study
how the introduction of the participation margin changes the way a technology shock is transmitted to the economy compared to other two cases: frictionless labor market with endogenous participation; matching frictions with exogenous participation. We showed that the interaction between matching frictions, participation decision, nominal rigidities and monetary policy crucially depend on the role of search costs.
References


A Value of Employment

Let us rewrite utility recursively:

$$U_t = \log(C_t) + \phi \frac{(1 - E_t - \Gamma(N_t - E_t))^{1+\nu}}{1+\nu} + \beta E_t \{U_{t+1}\} \quad (A.1)$$

We want to compute $\frac{\partial U_t}{\partial E_t}$ taking into account (2.3) and (2.5):

$$\frac{\partial U_t}{\partial E_t} = \frac{1}{C_t} \left[ \frac{W_t}{P_t} - b \right] - \phi h_t^\nu (1 - \Gamma) + \beta E_t \left\{ \frac{\partial U_{t+1}}{\partial E_t} \right\} \quad (A.2)$$

Note that:

$$\frac{\partial U_{t+1}}{\partial E_t} = (1 - \rho)(1 - f_{t+1}) \left[ \frac{1}{C_{t+1}} \left( \frac{W_{t+1}}{P_{t+1}} - b \right) - \phi h_{t+1}^\nu (1 - \Gamma) + \beta E_{t+1} \left\{ \frac{\partial U_{t+2}}{\partial E_{t+1}} \right\} \right]$$

$$= (1 - \rho)(1 - f_{t+1}) \frac{\partial U_{t+1}}{\partial E_{t+1}} \quad (A.3)$$

therefore we can rewrite (A.2) as:

$$\frac{\partial U_t}{\partial E_t} = \frac{1}{C_t} \left[ \frac{W_t}{P_t} - b \right] - \phi h_t^\nu (1 - \Gamma) + E_t \left\{ (1 - \rho)(1 - f_{t+1})\beta \frac{\partial U_{t+1}}{\partial E_{t+1}} \right\} \quad (A.4)$$

Let $V_t^w := \frac{\partial U_t}{\partial E_t}/U_{c,t} = \frac{\partial U_t}{\partial E_t} C_t$ be the surplus from employment in terms of current consumption of the final good. Then,

$$V_t^w = \frac{W_t}{P_t} - b - \phi h_t^\nu (1 - \Gamma) C_t + E_t \left\{ (1 - \rho)(1 - f_{t+1})\beta \frac{C_t}{C_{t+1}} V_{t+1}^w \right\} \quad (A.5)$$

that coincides with equation (2.15) in the text.
Figure 1: Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous). Parametrization: high workers’ bargaining power - $\eta = 0.05$ - and flexible prices.
Figure 2: Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous). Parametrization: high workers' bargaining power - \( \eta = 0.05 \) - and sticky prices.
Figure 3: Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous). Parametrization: $\eta = 0.5$ and flexible prices.
Figure 4: Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous). Parametrization: $\eta = 0.5$ and sticky prices.
Figure 5: Impact Responses as a function of the inflation reaction coefficient $\phi_p$ in the monetary policy rule. Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous). Parametrization: $\eta = 0.05$
Figure 6: Impact Responses as a function of the inflation reaction coefficient $\phi_p$ in the monetary policy rule. Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous). Parametrization: $\eta = 0.5$