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Commodity tax competition and industry location under the destination- and the origin-principle

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(Revised version)

Abstract

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Keywords: commodity tax competition; origin principle; destination principle; tax harmonization; industry location

JEL Classification: F12; H22; H87; R12

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1 Introduction

Increases in factor mobility and economic integration have given rise in the European Union (henceforth, EU) to concerns about the possibilities of tax competition eroding governments’ ability to finance public expenditure and to maintain the welfare state. Devolution of responsibility for public services to regions within the EU has arguably further increased opportunities for harmful tax competition, both within and between countries. In the USA, where state and local taxes support several important categories of public expenditure, concern about tax competition has a much longer history but remains, nevertheless, a contentious issue. In recent years, one added source of concern in both the EU and the USA has been the development of e-commerce, which has exacerbated already existing pressures on cross-border tax systems (Goolsbee, 2001). Furthermore, in the case of the EU, increasing economic integration might be an important driver for regional inequalities, and these inequalities may be further amplified by tax competition. Since one of the social cohesion objectives of the EU, as explicitly spelled out by Article 130a of the Amsterdam Treaty of 1997, is to “aim at reducing disparities between the levels of development of the various regions and the backwardness of the least favoured regions or islands, including rural areas”, tax competition may have additional indirect costs if it were to accelerate regional divergence, thus leading to the increased use of structural funds to reverse such an undesirable evolution.¹

While much of the tax competition literature focuses on capital or corporate profit taxes, there are important tax competition issues that arise for value-added taxes (henceforth, VAT) and retail sales taxes when consumers can shop in multiple jurisdictions. It is generally thought that there are two key questions in the design of indirect taxation systems with cross-border transactions (Keen et al., 2002).² The first question asks where taxes should be levied. Two alternative regimes are traditionally considered. Under the ‘destination or consumption-based principle’ (henceforth, DP) tax is paid in the country where goods are consumed at the rate applied there, while under the ‘origin or production-based principle’ (henceforth, OP) tax is paid in the country where goods are produced at that country’s rate. Accordingly, local consumption is taxed and exports are exempted under DP, while local production is taxed and imports are exempted under OP. Once a tax principle has been agreed on, the second question asks whether tax rates should be set

¹The EU allocates roughly 35% of its budget to the European Regional Development Funds (195 million euros for the 2000–2006 period). Of these, almost 70% go to so-called ‘Objective 1 regions’, which are defined as those regions with an average income of less than 75% of the EU average.

²Our spatial setting differs markedly from the usual models of commodity tax competition focusing on cross-border shopping. Here, we consider a shipping model in which goods are traded between countries on segmented markets, and in which firms choose where to locate and to produce (instead of consumers choosing where to shop).
independently by national governments or rather 

harmonized to some extent across countries. While the former case allows for more flexibility in dealing with asymmetric shocks and entices governments to exert effort to collect taxes efficiently, it may make them engage in harmful tax competition.

Within the EU, the choice of a tax principle and the question of VAT harmonization are a recurrent source of debate (European Commission, 2000; Ebrill et al., 2001). In particular, while “the concept of a definitive system of taxation in the Member State of origin [is] retained as a long-term Community objective” (European Commission, 2004b, p.1), with increasing direct cross-border sales to consumers such a system for all types of transactions (partly motivated by the relative administrative ease for collecting and remitting tax revenue) has been met with skepticism by many countries which fear losing production and tax revenue. In the USA, a closely related important issue has been whether or not to close the use tax evasion loophole that puts increasing strain on local governments’ budgets. The Streamlined Sales Tax Project (SSTP) is an attempt at getting cooperation among states in taxation of cross-border purchases. This proposal would result in DP treatment of mail-order and online purchases, and OP treatment of purchases by consumers who travel out-of-state. As in the EU, this reform proposal has not met with broad agreement, as shown by state governments’ chronic delays in implementing the necessary regulatory requirements (Strayhorn, 2005).

The main objective of this paper is to analyze commodity tax competition, tax harmonization, and industry location in a framework featuring internationally mobile firms, imperfect competition, and asymmetric country sizes. To do so, we rely on the model developed by Behrens et al. (2007), which involves two sectors, one supplying a homogeneous good under perfect competition while the other supplies a differentiated good under by monopolistic competition. Our framework generates a linear demand system with variable mark-ups. Such a setting allows us to capture the various impacts that tax incidence and trade costs may have on the equilibrium tax rates and the spatial distribution of industry. We push further the analysis developed by Behrens et al. (2007) since we no longer assume that tax rates are exogenously given. Instead, we consider a simple tax competition game where two jurisdictions in a federation each maximize the sum of its residents’ consumer surplus, returns to locally-owned capital, and the benefits of local public expenditure. Within this framework, we compare the equilibrium outcomes under both DP and OP to two different cooperative outcomes. In the first, the federation chooses independently tax rates in both jurisdictions to maximize total welfare (‘cooperative outcome’); in the second, the

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3 Recent experience with extending existing agreements on reduced VAT rates in construction and hospitality has shown that agreeing on tax matters is likely to get even more difficult in the enlarged EU.

4 Every state that levies a retail sales tax also levies a use tax at the same rate, which applies to goods purchased out-of-state. Enforcement is difficult since the Supreme Court has ruled that out-of-state retailers cannot be compelled to collect these taxes unless the firms ‘have nexus’, i.e., do some business within the purchaser’s state of residence.
federation chooses a single harmonized tax rate with the same objective (‘harmonized outcome’). We also deal with the question of how a switch from DP to OP for final consumption transactions, as initially envisaged by the European Commission (2004b), may change the equilibrium. In so doing, we pay particular attention to the impacts of a spatial redistribution of industry entailed by such a regime change. More precisely, we analyze how equilibria differ according to whether or not industry location reacts to changes in tax rates and principle.

There is a long literature on the possibly different impacts of DP and OP under VAT. Considerable research focuses on when the two principles are equivalent in the sense that a switch from one to the other has no real effects (see Lockwood et al., 1994, for details on equivalence results). These models all incorporate exchange rate adjustments and balanced trade, so that relative prices do not change when VAT rates are uniform across goods and there is no factor mobility. Our model instead considers taxation only in the differentiated sector (so that there is an untaxed good), while exchange rates do not adjust as within the Euro zone or the USA. Furthermore, we study the effects of a change in tax rates on the spatial distribution of industry. We thus explicitly take into account the fact that a switch from a ‘consumption tax’ (DP), which falls mostly on immobile consumers, to a ‘production tax’ (OP), which falls mostly on mobile firms, may have strong spatial effects in the long run as firms react to changes in tax systems, thereby enticing governments to engage in more tax competition.

Our findings confirm this intuition by showing that the ‘race to the bottom’ is particularly strong under OP, thus leading to quite low non-cooperative equilibrium tax rates. By contrast, we will see that there is little fiscal competition under DP. Consequently, the gains from either unified or harmonized tax setting are much larger under OP than under DP. Our results further show that the larger country charges a higher tax rate than the smaller one regardless of the tax principle. Since under OP the higher tax rate in the larger market works to raise the firms’ production costs when compared to the smaller market, in equilibrium fewer firms will be attracted to the larger country than under DP. Consequently, the so-called ‘home market effect’, which states that the larger country attracts a disproportionate share of firms (Helpman and Krugman, 1985), gets attenuated under OP and there will be a more balanced industry distribution than under DP. A direct policy implication of this finding is that, in the absence of tax harmonization, federations like the EU face a non-trivial trade-off: more spatial inequality and more tax revenue under DP, or less spatial inequality and less tax revenue under OP. Because a quite substantial part of the EU’s structural funds, intended to alleviate spatial inequality and backwardness, are financed indirectly by national VAT revenues, the question of whether and how VAT rates and VAT regimes may exacerbate these inequalities in the first place deserves closer attention. We may then safely conclude that fiscal competition has drastically different effects under the destination principle than under the origin principle, especially in the long run when industry location is endogenous.

The remainder of the paper is organized as follows. Section 2 presents the model and derives
the market outcome. Section 3 then deals with the short-run equilibrium and optimum of the tax game, both under DP and OP, taking the spatial distribution of industry as given. We also discuss the harmonized outcome. Section 4 turns to the long-run equilibrium with internationally mobile firms. The downside of using such a rich modeling framework as we do is that analytical results for the tax competition equilibria with firm mobility are largely out of reach. We, therefore, rely on numerical analysis to highlight the model’s main properties. We investigate numerically how a switch from DP to OP affects equilibrium tax rates and the spatial distribution of industry, and we discuss the harmonized outcome, taking into account firm mobility. Section 5 points to the policy relevance of our main findings and concludes.

2 The model

Consider an economy with two countries, labeled $H$ (home) and $F$ (foreign), and a unit total mass of consumers. The units of labor and of capital are chosen such that each consumer is endowed with one unit of labor and one unit of capital. Let $0 < \theta < 1$ (resp., $1 - \theta$) denote the share of consumers located in country $H$ (resp., $F$), which implies that $\theta$ (resp., $1 - \theta$) also measures that country’s shares of labor and capital. Consumers are internationally immobile and supply labor only in the country where they reside. By contrast, they are free to supply capital wherever they want and do so seeking the highest (nominal) rate of return. These two contrasting assumptions on factor mobility aim to capture the fact that, in the real world, capital is much more mobile than labor.

2.1 Preferences and technology

All consumers have identical quasi-linear preferences over a homogenous good and a continuum of varieties of a horizontally differentiated good with measure $N$. The subutility over the varieties of the differentiated good is quadratic as in Ottaviano et al. (2002). A resident of country $i = H, F$ solves the following consumption problem:

$$\max_{q_i(v), \ v \in [0, N]} U_i \equiv \alpha \int_0^N q_i(v)dv - \frac{\beta - \gamma}{2} \int_0^N [q_i(v)]^2dv - \frac{\gamma}{2} \left[ \int_0^N q_i(v)dv \right]^2 + Z_i$$

s.t. $$\int_0^N p_i(v)q_i(v)dv + p_i^Z Z_i = R_i + w_i + p_i^Z Z_0$$

where $q_i(v)$ and $p_i(v)$ are the individual demand and the (consumer) price of variety $v$; $Z_i$ and $p_i^Z$ are the individual demand and the price of the homogeous good; $R_i$ is the return to the agent’s unit of capital; $w_i$ is the wage rate in country $i$; $Z_0$ is a sufficiently large initial endowment of the homogeneous good; and $\alpha > 0$, $\beta > \gamma > 0$ are parameters.

5In this respect, our model offers a better description of the EU, in which interregional labor mobility is much lower than in the USA (Faini, 1999).
The homogeneous good is produced under constant returns to scale and perfect competition by using one unit of labor per unit of output. We assume that it can be traded freely. Due to perfect competition and free trade in that good, profit maximization then implies that \( p_i^Z = w_i = 1 \) in both countries, where the last equality is our choice of numéraire. By contrast, each variety of the differentiated good is produced under firm-level increasing returns to scale and monopolistic competition, with a fixed requirement \( \phi \) of capital and a constant marginal requirement \( m \) of labor. Because there is a unit mass of capital, capital market clearing then implies that the total mass of firms is determined by \( N = 1/\phi \). Since we work with a continuum of varieties of a fixed mass, we may choose units such that \( N = 1 \), thus implying \( \phi = 1 \) (Baldwin et al., 2003).

Throughout the paper, we alleviate notation by presenting expressions for country \( H \) only. The expressions associated with country \( F \) are symmetric and are obtained by switching the \( H \) and \( F \) subscripts. Because all goods produced in the same country enter consumers’ utility functions symmetrically, we may drop the variety index \( v \) in what follows.

Denote by \( 0 < \lambda < 1 \) the share of firms (and varieties) located in country \( H \). The corresponding share in country \( F \) is given by \( 1 - \lambda \). Letting \( q_{HH} \) (resp., \( q_{FH} \)) be the consumption in country \( H \) of a variety produced in country \( H \) (resp., \( F \)), and \( p_{HH} \) (resp., \( p_{FH} + \tau \)) the mill (resp., the delivered) consumer price in country \( H \) for a variety produced in country \( H \) (resp., \( F \)), utility maximization yields the following demand functions:

\[
q_{HH} = a - (b + c)p_{HH} + cP_H
\]

\[
q_{FH} = a - (b + c)(p_{FH} + \tau) + cP_H
\]

where \( a \equiv \alpha/\beta \) expresses the desirability of the differentiated product with respect to the numéraire; \( b \equiv 1/\beta \) gives the link between individual and industry demands (consumers become more sensitive to price differences when \( b \) rises); \( c \equiv \gamma/\beta(\beta - \gamma) \) is an inverse measure of the degree of product differentiation between varieties (when \( c \to \infty \), varieties are perfect substitutes, whereas they are independent for \( c = 0 \)); and where

\[
P_H \equiv \lambda p_{HH} + (1 - \lambda)(p_{FH} + \tau)
\]

stands for the average consumer price of the differentiated good in country \( H \). Using expressions (1) and (2), the consumer surplus in country \( H \) can then be expressed as follows:

\[
C_H = \frac{a^2}{2b} - a \left[ \lambda p_{HH} + (1 - \lambda)(p_{FH} + \tau) \right]
+ \frac{b + c}{2} \left[ \lambda (p_{HH})^2 + (1 - \lambda)(p_{FH} + \tau)^2 \right] - \frac{c}{2} \left[ \lambda p_{HH} + (1 - \lambda)(p_{FH} + \tau) \right]^2.
\]
2.2 Price equilibrium

In what follows, we superscript all relevant variables with \(d\) and \(o\) under DP and OP, respectively. A firm located in country \(H\) maximizes its profit given by

\[
\Pi_H \equiv \theta [p_{HH}(1-t) - m]q_{HH} + (1-\theta)[p_{HF}(1-\tilde{t}) - m]q_{HF} - r_H
\]

where \((t, \tilde{t}) = (t^d_H, t^d_F)\) under DP, and where \((t, \tilde{t}) = (t^o_H, t^o_F)\) under OP. We rewrite the profit functions using a device that allows us to treat an ad valorem tax as a specific tax and a pure profits tax. Letting \(s \equiv t/(1-t)\) and \(\bar{s} \equiv \tilde{t}/(1-\tilde{t})\), the profit function can be rewritten as:

\[
\Pi_H = \frac{1}{1+s} \theta [p_{HH} - m(1+s)]q_{HH} + \frac{1}{1+\bar{s}} (1-\theta)[p_{HF} - m(1+\bar{s})]q_{HF} - r_H
\]

which is our working specification. In what follows, we let \((s, \bar{s}) = (s^d_H, s^d_F)\) under DP and \((s, \bar{s}) = (s^o_H, s^o_F)\) under OP.

As mentioned in the introduction, national product markets are segmented.\(^7\) Workers being spatially immobile, labor markets are local and wages are country-specific. Each firm, therefore, maximizes its domestic and foreign profits by maximizing each term in expression (3) independently. Since each firm is negligible to the market because of the continuum assumption, it takes the price index (2) as given. Yet, firms’ pricing decisions remain interdependent ‘in the aggregate’, because each firm must correctly anticipate what the average price of the market will be in equilibrium. Thus, our price equilibrium is formally equivalent to a Nash equilibrium with a continuum of players.

As shown by Behrens et al. (2007), the Nash equilibrium (producer) prices in the absence of commodity taxes are given by

\[
p^*_HH = \frac{2[a + (b+c)m] + c(1-\lambda)\tau}{2(2b+c)}
\]

\[
p^*_FH = \frac{2[a + (b+c)m] + c(1-\lambda)\tau}{2(2b+c)} - \frac{\tau}{2}
\]

The equilibrium prices under DP and under OP can then be conveniently expressed with respect to the no-tax case as follows:

\[
p_{HH}^d = p_{HH}^* + \frac{b+c}{2b+c} ms_{H}^d
\]

\[
p_{FH}^d = p_{FH}^* + \frac{b+c}{2b+c} ms_{H}^d
\]

\(^6\)Note that \(t\) and \(s\) are monotonic increasing transformations. Increasing \(t\) (resp., \(\tilde{t}\)) thus amounts to increasing \(s\) (resp., \(\bar{s}\)).

\(^7\)See Engel and Rogers (1996) and Haskel and Wolf (2001) for empirical evidence on market segmentation. Segmented markets are usually neglected in the existing literature building on the CES model, since price discrimination is irrelevant with identical iso-elastic demands (e.g., Haufler and Pflüger, 2004).
Expressions (4) and (5) are tax-inclusive (consumer) price s net of trade costs. Using the foregoing equilibrium prices, the equilibrium individual demands under the two tax principles can be rewritten as follows:

\[ q^d_{HH} = (b + c) \left[ p^*_{HH} - m(1 + s^d_H) \right] \]
\[ q^d_{FH} = (b + c) \left[ p^*_{FH} - m(1 + s^d_H) \right] \]

and

\[ q^o_{HH} = (b + c) \left[ p^o_{HH} - m(1 + s^o_H) \right] \]
\[ q^o_{FH} = (b + c) \left[ p^o_{FH} - m(1 + s^o_H) \right] . \]

Throughout this paper, we assume that marginal production costs \( m \) and trade costs \( \tau \) are sufficiently low for international demands \( q_{HF} \) and \( q_{FH} \) to remain strictly positive for all firm distributions \( \lambda \) under both tax principles. This is always the case if trade costs \( \tau \) and marginal costs \( m \) are low enough (see Appendix A.1 for a more precise statement of the conditions).

### 2.3 Spatial equilibrium

A spatial equilibrium is such that no agent has an incentive to change her international allocation of capital and such that no firm has an incentive to enter or exit the market. These two conditions will hold when no agent can get a higher rental rate by relocating her capital and when rental rates exactly absorb firms’ operating profits. Formally, at an interior spatial equilibrium \((0 < \lambda^* < 1)\), we have \( r_H = r_F = r \) with \( r \) such that the profits (3) are equal to zero. In what follows, we restrict ourselves to such equilibria. This does not seem too stringent in practice, especially if one has in mind an aggregate sectoral structure of the economy.

Substituting the equilibrium quantities into the profits (3), the rental rates under DP equate profits to zero and are as follows:

\[ r^d_H = \theta \frac{b + c}{1 + s^d_H} \left[ p^d_{HH} - m(1 + s^d_H) \right]^2 + (1 - \theta) \frac{b + c}{1 + s^d_F} \left[ p^d_{FH} - m(1 + s^d_F) \right]^2 \]  
(6)

The rental rates under OP are obtained in an analogous way and are given by:

\[ r^o_H = \theta \frac{b + c}{1 + s^o_H} \left[ p^o_{HH} - m(1 + s^o_H) \right]^2 + (1 - \theta) \frac{b + c}{1 + s^o_F} \left[ p^o_{FH} - m(1 + s^o_F) \right]^2 \]  
(7)

As shown by Behrens et al. (2007), under DP, we can readily determine the spatial equilibrium by solving the equation \( r^d_H = r^d_F \) (see Appendix A.2 for an expression of the solution). Under OP, the spatial equilibrium is a solution to a quadratic equation that is too complex to allow for a detailed analytical investigation with positive trade costs.
2.4 Capital and tax revenues

In what follows, we examine the impacts of tax competition and tax harmonization on welfare and equilibrium tax rates in two different cases. First, we will examine the short run in which the spatial distribution of firms $\lambda$ does not change. Second, we will turn to the long run in which the spatial distribution of firms does react to changes in tax rates. Note that since capital does not relocate to equalize rental rates in the short run, country-specific rates of returns may differ. In that case, aggregate claims to capital income in both countries will depend on how agents in each country allocate their capital across countries. To cut short a taxonomy of cases that arise under competing assumptions on the distribution of capital incomes, we assume that each agent holds a fully diversified portfolio. Put differently, all agents have claims to an equal share of world profits.\(^8\) This assumption implies that even in the short run the capital revenues $R_H$ and $R_F$ are equal and given by

$$R_H = R_F \equiv \lambda r_H + (1 - \lambda)r_F.$$  

In the long run, when capital moves across countries to equalize rental rates, we have of course $R_H = R_F = r$ at any interior equilibrium. Yet, one should keep in mind that even then the precise ownership pattern will influence the tax competition game.

Tax revenues associated with the tax rates $s^d_H$ and $s^d_F$ that accrue to the government of $H$ under DP are given by:

$$T^d_H = \theta \frac{s^d_H}{1 + s^d_H} \left[ \lambda p^d_H q^d_H + (1 - \lambda) p^d_F q^d_F \right]$$  

whereas under OP they can be expressed as follows:

$$T^o_H = \lambda \frac{s^o_H}{1 + s^o_H} \left[ \theta p^o_H q^o_H + (1 - \theta) p^o_F q^o_F \right].$$

Expressions (8) and (9) clearly show that a commodity tax under DP corresponds to a consumption tax, which is thus proportional to market size ($\theta$); whereas a commodity tax under OP corresponds to a production tax, which is thus proportional to industry size ($\lambda$).

3 Short-run equilibrium and optimum

We start by analyzing the case where governments take the spatial distribution of the industry as given. This may be either due to the fact that capital is relatively immobile between countries,

\(^8\)This assumption is not fully satisfying as empirical evidence highlights the existence of a significant home bias in international equity holdings (e.g., French and Poterba, 1991; Ahearne et al., 2004). Dealing with distributional issues related to international portfolio diversification (or its absence!) is a complex issue in models where the allocation of capital is endogenous and where governments set taxes non-cooperatively. To the best of our knowledge, there does not exist to date a model of economic geography dealing with home bias in capital holdings. We thus stick with the assumption of full portfolio diversification.
or because governments do not realize that changing their commodity tax rates may have an influence on the spatial structure of the economy. This short-run analysis provides a benchmark against which we can judge the outcome when firms’ location choices are taken into account. Doing so allows us to highlight the relative importance of factor mobility when deciding on a tax principle or on tax harmonization in the long run, an aspect developed more fully in Section 4 below.

3.1 The destination principle

Given our quasi-linear utility function, social welfare in each country may be expressed as the sum of consumers’ surplus, firms’ profits, and utility derived from local public goods financed by local tax revenues. Because of our assumption of full portfolio diversification, and recalling that wages are equal to one, welfare in country \( H \) is given as follows:

\[
W^d_H = \theta \left[ C^d_H + 1 + \lambda r^d_H + (1 - \lambda) r^d_F \right] + 2\theta \left( \frac{T^d_H}{2\theta} \right)^{1/2}.
\]

The last term in the foregoing expression denotes the utility derived from the consumption of a quasi-private public good. Given that \( \lambda \) is fixed in the short run, the non-cooperative tax equilibrium is a solution to the first-order conditions \( \partial W^d_H / \partial s^d_H = \partial W^d_F / \partial s^d_F = 0 \), where

\[
\frac{\partial W^d_H}{\partial s^d_H} = \theta \left[ \frac{\partial C^d_H}{\partial s^d_H} + \lambda \frac{\partial r^d_H}{\partial s^d_H} + (1 - \lambda) \frac{\partial r^d_F}{\partial s^d_H} \right] + \frac{1}{2} \frac{\partial T^d_H}{\partial s^d_H} \left( \frac{T^d_H}{2\theta} \right)^{-1/2} = 0.
\]

All expressions for the individual components of the derivatives are relegated to Appendices A.3 and A.4. Although the analytical expressions are cumbersome, numerical simulations reveal that \( W^d_H \) is concave in \( s^d_H \) for any given value of \( s^d_F \), which then yields a single-valued reaction function and (in our model) a unique equilibrium. In what follows, we denote by \( s^d_{H^*} \) and \( s^d_{F^*} \) the non-cooperative equilibrium tax rates under short-run DP.

The cooperative equilibrium tax rates, henceforth denoted by \( s^{dc}_H \) and \( s^{dc}_F \), are such that world welfare \( W^d \equiv W^d_H + W^d_F \) is maximized with respect to \( s^d_H \) and \( s^d_F \). Under DP with segmented markets, because foreign taxes have no direct impact on home prices, we have \( \partial C^d_H / \partial s^d_F = \partial T^d_H / \partial s^d_F = 0 \). The cooperative equilibrium is then a solution to the first-order conditions

\[9\text{The population of each country being fixed, maximizing per capita welfare or total welfare is equivalent in the tax game.}\]

\[10\text{Because of decreasing marginal utility of tax revenue, the assumption of a quasi-private public good is preferable to the assumption of a pure public good which would favor the larger country (Wilson, 1991). We thank a referee for suggesting this point. Assuming that the utility derived from the public good is a linear function of } T \text{ does not reduce the complexity of our model, yet generates additional difficulties because equilibrium tax rates could be zero (or even negative, i.e., consumption subsidies, as in Haufner and Pflüger, 2004) unless tax revenues enter with a sufficiently large weight (see, e.g., Laffont and Tirole, 1993).}\]
\[ \frac{\partial W_d}{\partial s_H} = \frac{\partial W_d}{\partial s_F} = 0, \] where
\[ \frac{\partial W_d}{\partial s_H} = \frac{\partial W_H}{\partial s_H} + (1 - \theta) \left[ \lambda \frac{\partial W_H}{\partial s_H} + (1 - \lambda) \frac{\partial W_F}{\partial s_H} \right] = 0 \] (10)

Since the tax derivatives of the DP rental rates are all negative (see Appendix A.4.), it is readily verified that
\[ \frac{\partial W_d}{\partial s_H}(s_H^{d*}, s_F^{d*}) < \frac{\partial W_H}{\partial s_H}(s_H^{d*}, s_F^{d*}) = 0 \]
which establishes the following result.

**Proposition 1 (short-run ‘race to the top’) ** When \( \lambda \) is fixed and agents have identical profit claims, the non-cooperative tax rates \( s_H^{d*} \) and \( s_F^{d*} \) under the destination principle are higher than the cooperative tax rates \( s_H^{c*} \) and \( s_F^{c*} \).

The intuition underlying the result in Proposition 1 is as follows. When country \( i \) changes its tax rate, it directly influences the profits of firms located in the other country. Part of this change feeds back into the income of country-\( i \) agents, an effect that is taken into account by the tax-setter in country \( i \). Yet, the negative externality inflicted upon country-\( j \) agents is disregarded. Proposition 1 establishes that in the non-cooperative equilibrium this negative externality dominates the positive one, so that both countries would benefit from a coordinated reduction of tax rates.\(^{11}\) The excess degree of equilibrium tax rates, when compared to optimum tax rates, depends on the countries’ sizes. We can show the following result.

**Proposition 2 (size and excess rates) ** When \( \lambda \) is fixed, the more similar the two countries are in terms of size, the larger the welfare gains from a coordinated reduction in tax rates under the destination principle.

**Proof.** Using the expressions of the rental rate derivatives, as given in Appendix A.4, some standard computations show that condition (10) and its counterpart for country \( F \) can be rewritten as follows:
\[ \frac{\partial W_d}{\partial s_H} = \frac{\partial W_H}{\partial s_H} + \theta(1 - \theta) [\lambda K_1 + (1 - \lambda) K_2] = 0 \]
\[ \frac{\partial W_d}{\partial s_F} = \frac{\partial W_F}{\partial s_F} + \theta(1 - \theta) [\lambda K_3 + (1 - \lambda) K_4] = 0 \]
where \( K_i, \) for \( i = 1, 2, 3, 4, \) are negative bundles of parameters that do not depend on \( \theta. \) Since \( \theta(1 - \theta) \) is maximal when \( \theta = 1/2, \) we may conclude that the welfare gains from a coordinated reduction in tax rates are largest when \( \theta = 1/2. \) \( \blacksquare \)

Proposition 2 shows that the fiscal externality is strongest when both countries are of roughly equal size. It is weakest when \( \theta \) is close to 0 or 1, as most of the externality is then internalized by the large country.

\(^{11}\)Note that an international home bias in equity holdings would make our result even stronger since in the limit only the negative effect would prevail.
3.2 The origin principle

Analogously to DP, welfare in country $H$ under OP is given by:

$$W^o_H = \theta \left[ C^o_H + 1 + \lambda r^o_H + (1 - \lambda) r^o_F \right] + 2\theta \left( \frac{T^o_H}{2\theta} \right)^{1/2}.$$  

The non-cooperative equilibrium $s^*_{H}^o$ and $s^*_{F}^o$ under OP is a solution to the first-order conditions $\partial W^o_H / \partial s^o_H = \partial W^o_F / \partial s^o_F = 0$, where

$$\frac{\partial W^o_H}{\partial s^o_H} = \theta \left[ \frac{\partial C^o_H}{\partial s^o_H} + \lambda \frac{\partial r^o_H}{\partial s^o_H} + (1 - \lambda) \frac{\partial r^o_F}{\partial s^o_H} \right] + \frac{1}{2} \frac{\partial T^o_H}{\partial s^o_H} \left( \frac{T^o_H}{2\theta} \right)^{-1/2} = 0.$$

Numerical simulations again reveal that $W^o_H$ is concave in $s^o_H$ for any given value of $s^o_F$, which then yields a single-valued reaction function and (in our model) a unique equilibrium.

The cooperative equilibrium tax rates, denoted by $s^c_{H}$ and $s^c_{F}$, are obtained as before by maximizing world welfare $W^o \equiv W^o_H + W^o_F$. Computing $\partial W^o / \partial s^o_H$, and making use of the first-order conditions for the non-cooperative equilibrium, we have

$$\frac{\partial W^o}{\partial s^o_H} (s^o_H, s^o_F) = (1 - \theta) \left[ \frac{\partial C^o_F}{\partial s^o_H} + \lambda \frac{\partial r^o_H}{\partial s^o_H} + (1 - \lambda) \frac{\partial r^o_F}{\partial s^o_H} \right] + \frac{1}{2} \frac{\partial T^o_F}{\partial s^o_H} \left( \frac{T^o_F}{2\theta} \right)^{-1/2}$$

where the right-hand sides are evaluated at the non-cooperative equilibrium tax rates. Contrary to DP, as discussed in the foregoing, we can no longer clearly sign the residual terms. This is because we have

$$\frac{\partial C^o_F}{\partial s^o_H} < 0 \quad \frac{\partial r^o_H}{\partial s^o_H} < 0 \quad \text{and} \quad \frac{\partial T^o_F}{\partial s^o_H} > 0 \quad \frac{\partial r^o_F}{\partial s^o_H} > 0.$$  

The first term captures the negative externality stemming from tax exporting under OP. The second term shows that higher taxes in $H$ reduce operating profits of firms located there, since the firms’ effective marginal cost (which includes the tax) increases in all markets. The third term shows that an increase in $s^o_H$ raises operating profits of firms located in $F$. The reason is that higher effective marginal cost in $H$ shifts market shares to firms in $F$, thus allowing them to pay higher returns to capital. Finally, the last term shows that tax revenue in $F$ increases with $s^o_H$, since firms in $F$ now sell more and at a higher price. Obviously, the total effect can go either way so that we cannot sign the externality. Stated differently, under the short-run OP the non-cooperative tax rates may a priori be either too low or too high with respect to the ones chosen in the cooperative outcome. The reason for the possible presence of a ‘race to the bottom’, even when firms are immobile, is that under OP an increase in the tax rate is formally equivalent to an increase in firms’ production costs. Thus, governments have an incentive to cut
rates to make domestic firms more competitive in the international market, thereby diverting tax revenue from the other country.\textsuperscript{12}

Although the reversal in the rankings of equilibrium and optimum tax rates does not allow for clear-cut results, we can show that it is linked to basic parameters of the model. In particular, large values of \( a \) and low values of \( c \) lead to short-run non-cooperative tax rates that exceed the ones that would be chosen in the cooperative equilibrium. Stated differently, when varieties are sufficiently differentiated (small \( c \)) or when preferences for the differentiated good are strong (large \( a \)), the short-run equilibrium tax rates will be inefficiently large. The intuition underlying this result is as follows. When \( a \) is large or when \( c \) is small, firms price in the inelastic portion of their demand. Since the rate of tax pass-through increases with \( a \) and \( c \) under OP (Behrens \textit{et al.}, 2007), this raises consumer prices (when \( c \) becomes small enough, the pass-through becomes almost 100\%). Hence, the negative effect of a tax increase on both home and foreign consumer surplus comes to dominate the positive effect of such an increase on tax revenues, which then makes the optimal rates fall with respect to the equilibrium rates.

### 3.3 Comparing the equilibrium rates

Note first that consumer surplus is affected by tax rates only through consumer prices, regardless of the tax principle. As shown in Appendix A.3, prices change as follows with tax rates:

\[
\begin{align*}
\frac{\partial p^d_{HH}}{\partial s^d_H} &> \frac{\partial p^o_{HH}}{\partial s^o_H} > 0 \\
\frac{\partial p^o_{HH}}{\partial s^o_F} &> \frac{\partial p^d_{HH}}{\partial s^d_F} = 0 \\
\frac{\partial p^o_{FH}}{\partial s^o_H} &> \frac{\partial p^d_{FH}}{\partial s^d_F} > 0
\end{align*}
\]

As one can see, consumer prices set by firms in country \( H \) are more sensitive to increases in local tax rates under DP than under OP, whereas the reverse holds with respect to consumer prices set by firms in country \( F \). Thus, consumer surplus in country \( H \) is more sensitive to increases in its own tax rate under DP than under OP, whereas the surplus externality imposed upon the other region is larger under OP than under DP:

\[
\frac{\partial C^d_H}{\partial s^d_H} < \frac{\partial C^o_H}{\partial s^o_H} < 0 \quad \text{and} \quad \frac{\partial C^o_F}{\partial s^o_H} < \frac{\partial C^d_F}{\partial s^d_H} < 0.
\]

The reason is that under DP the tax is a consumption tax, whereas under OP the tax is a production tax that can be partly exported to the foreign consumers. Since the externality is stronger under OP than under DP, one might a priori suspect that non-cooperative OP rates should exceed non-cooperative DP rates. However, higher taxes under OP in one country make all firms located in that country relatively less competitive (formally, their marginal cost increases),

\textsuperscript{12}Changing tax rates under OP is equivalent to changing firms’ marginal costs. Formally, tax competition under OP is similar to export subsidizing and thus closely related to strategic trade policy considerations (Brander, 1995).
thereby reducing profits and possibly tax revenues as sales drop in all markets relative to the now more competitive foreign firms. Whether OP rates thus exceed DP rates in the short run is unclear. To obtain additional insights, we can verify that

\[
\frac{\partial^2 C^d_H}{\partial s^d_H \partial a} < \frac{\partial^2 C^o_H}{\partial s^o_H \partial a} < 0,
\]

which reveals that the differential sensitivity of consumer surplus with respect to taxes between the two regimes actually rises with the value of \(a\). Consequently, under OP the marginal effect on consumer surplus of raising taxes is lower than the corresponding effect under DP for high values of \(a\), which implies that tax rates will be higher in the former case than in the latter. Furthermore, a higher value of \(a\) implies that consumers have stronger preference for the differentiated goods, thereby making demands less elastic. This dampens the negative effect on domestic firms’ competitive position associated with higher OP taxes, thus raising equilibrium OP tax rates. Hence, the short-run OP rates may actually exceed the short-run DP rates for sufficiently high values of \(a\).

4 Numerical analysis of the long-run equilibrium

We now relax the restrictive assumption of a fixed spatial distribution \(\lambda\). Doing so is important for several reasons. First, it is well known that factor mobility has an important impact on non-cooperative equilibrium tax rates. Disregarding it may thus yield predictions that significantly differ from outcomes when firms are mobile. Second, tax competition and factor mobility may lead to an uneven spatial distribution of economic activities, which may pose problems on both efficiency and equity grounds in the EU. Since commodity taxation remains a national matter for now, non-cooperative tax setting is bound to have an impact on the spatial allocation of resources. It is, therefore, of interest to take into account these long-run effects when assessing the relative desirability of OP or DP, or the possible impacts of a regime change.

Unfortunately, we cannot pursue our analytical investigation beyond this point. We instead appeal to numerical simulations which we calibrate to yield equilibrium long-run DP tax rates in the VAT ranges currently used in the EU. In addition, we have chosen parameter values that yield plausible mark-ups and commodity flows. Our numerical results aim to illustrate and supplement our previous findings and to discuss the possible impacts of a regime change and of tax harmonization, both in the short-run when locations are fixed and in the long-run when firms may react to changing tax incentives.\(^\text{13}\) In the subsequent analysis, our baseline values for parameters are set as follows: \(\alpha = 1, \beta = 0.25, \gamma = 0.2\) and \(m = 0.7\). Since our primary interest is to investigate how different tax regimes affect the equilibrium outcomes in the short and long run when firms are mobile, countries are of different size, and trade is costly, we allow

\(^{13}\text{See Appendix B for a description of the methodology and its implementation.}\)
both trade costs $\tau$ and relative country sizes $\theta$ and $1-\theta$ to vary, while holding all other structural parameters fixed. Note that constraints on the values of $\theta$ and $\tau$ stem from our focus on interior equilibria ($0 < \lambda^* < 1$) in which there is bilateral trade. This places restrictions on the degree of asymmetry and the value of trade costs, which allow only for relatively small variations. We choose either $\tau = 0.05$ or $\tau = 0.07$ and $\theta \in \{0.5, 0.51, 0.52, 0.53, 0.54, 0.55\}$.

In what follows, we denote by $t^{ds}_H$ and $t^{ds}_F$ the equilibrium tax rates under DP, and by $t^{co}_H$ and $t^{co}_F$ the corresponding equilibrium tax rates under OP. Analogously, we let $t^{dc}_H$ and $t^{dc}_F$ denote the cooperative tax rates under DP, and $t^{oc}_H$ and $t^{oc}_F$ the cooperative tax rates under OP. Finally, $t^{dh}$ and $t^{oh}$ denote the single harmonized tax rates under the two tax principles.

### 4.1 The destination principle

Table 1 presents the equilibrium, the cooperative, and the harmonized tax rates under the long-run DP, as well as the corresponding equilibrium spatial distribution of firms across countries.\(^{14}\) For our baseline parameter set, *long-run equilibrium rates are always lower than either cooperative or harmonized rates*, but the two stay within two percentage points of each other. This suggests that capital mobility does not seem to lead to a vigorous race to the bottom under DP, which may explain why increases in capital and firm mobility in the EU have had little effect on national VAT rates to date. Note also that differences in market size have only relatively small effects on tax rates but, as known from the literature on the home market effect, large effects on industry location. As can be seen from Table 1, the larger country consistently charges higher equilibrium and optimum tax rates. The intuition for the former finding is that the larger market offers, ceteris paribus, a locational advantage for firms, which can serve a larger share of their demand locally and thus save trade costs. As firms agglomerate in the larger country, competition intensifies there and domestic prices fall, thus making consumer demands more elastic and enticing firms to absorb a larger part of the taxes. This in turn makes the government increase its tax rate since the additional tax absorption by firms mitigates the negative impact of increased commodity taxes on consumer surplus. In some sense, the trade cost savings from locating in the large market generate ‘agglomeration rents’, which can then be partly taxed away by the government via commodity taxes (see Baldwin and Krugman, 2004, for the case of capital taxes).

\(^{14}\)We report results for $\tau = 0.07$ only. Different values for trade costs yield qualitatively identical results. We point out later some effects of changes in $\tau$. 

Turning to the spatial allocation of firms, it is worth noting that $\lambda^{ds} > \lambda^{dh} > \lambda^{dc}$. In words, the spatial distribution of firms is skewed towards the larger country when taxes are set non-cooperatively, whereas it is less skewed in the cooperative case. Tax competition widens the tax
gap between regions and leads to more firms locating in the larger market, thereby exacerbating regional inequality. The non-cooperative outcome leads to excessive agglomeration in the large country when compared to either the cooperative or the harmonized outcome, thus suggesting that tax competition may be an additional cause of regional inequality.

Table 2 summarizes the welfare effects of non-cooperative tax setting under DP. As one can see, the relatively mild ‘race to the bottom’ does not lead to substantial welfare losses in the non-cooperative case. When compared to either the cooperative outcome or the harmonized outcome, welfare losses always fall short of 1%. Note that while both countries would lose from non-cooperative tax setting as compared to cooperative tax setting, the larger country would actually gain from tax harmonization. The increase in the tax rate for the smaller region raises welfare in the larger region as it is no longer at a tax disadvantage.

To summarize our key findings, tax competition does not seem to be a serious issue under DP even when firms are mobile and may relocate in response to differences in tax rates. The main reason is that DP VAT is basically a consumption tax which falls on immobile consumers and, therefore, does not stimulate much tax competition.

4.2 The origin principle

Table 3 presents the equilibrium, the cooperative, and the harmonized tax rates under the long-run OP, as well as the corresponding equilibrium spatial distribution of firms across countries. For our baseline parameter set, long-run equilibrium rates are always much lower than either cooperative or harmonized rates, as they fall to less than 2%. This suggests that capital mobility leads to a vigorous race to the bottom under OP. The main reason is that OP taxes are production taxes which directly affect firms’ marginal production costs for all markets (as they are fully levied in the region of production regardless of where subsequent consumption occurs). Governments then have a strong incentive to undercut each others rates in order to attract firms to raise more revenue and to lower consumer prices via trade cost savings. The strong opposition to recent proposals by the EU to switch to the origin principle appears to have a strong basis – competition for mobile capital would place a great strain on national budgets.

As one can further see from Table 3, the larger country again charges the higher equilibrium tax rates and the tax gap between countries increases with size asymmetries. The larger country offers, ceteris paribus, a more profitable production location, which generates agglomeration rents that the government can tax away. Moreover, tax rates in the cooperative outcome are such that the tax gap between the large and the small country are higher than under DP (compare with
Table 1). This larger tax gap forces a more equal distribution of firms under OP than under DP ($\lambda^{oc} < \lambda^{dc}$ and $\lambda^{oc*} < \lambda^{dc*}$). The larger tax gap under OP leads to less spatial inequality as more firms avoid locating in the high tax country.

[Insert Table 4 about here.]

The welfare effects of non-cooperative tax setting under OP are summarized in Table 4. As one can see, the strong race to the bottom has quite substantial welfare implications, with welfare falling about 9% when compared to the cooperative outcome. Tax competition under OP thus appears to have large social costs, thus suggesting that cooperation and/or harmonization are essential under an OP regime when firms are internationally mobile. Note, finally, that as under DP, while both countries would lose from non-cooperative tax setting as compared to cooperative tax setting, the larger country would actually gain from tax harmonization. As with DP, harmonization raises the tax rate in the smaller region, thereby benefiting the larger region due to a reduction in the tax gap.

4.3 Impacts of a regime change

How would a hypothetical regime change from DP to OP affect the tax equilibrium, both in the short run when capital is internationally immobile and in the long run when firms may relocate? Consider first the short run case. To begin with, note that the answer hinges on the spatial distribution $\lambda$ at which the regime change takes place. Since the initial choice of $\lambda$ is somewhat arbitrary (recall that $\lambda$ is just a parameter in the short run), it appears relevant to us to fix $\lambda$ at a ‘plausible’ value. We hence choose to hold the spatial distribution of firms constant at the spatial equilibrium that arises at the non-cooperative equilibrium tax rates under DP, i.e., $\lambda = \lambda^{dc*}$. Doing so allows us to interpret the short-run regime change as one in which the economy is initially in its long-run DP equilibrium, and in which governments consider changing to OP without taking into account the subsequent adjustment in $\lambda$. This case strikes us as a plausible scenario in a federation like the EU, where the spatial impacts of tax regime changes do not loom large (or even figure) on policy makers’ agendas.

Starting from $\lambda^{dc*}$ and holding this value constant, we solve for short-run equilibrium tax rates under OP. As seen in Table 5, while the regime change from DP to OP lowers rates, they do not fall dramatically. All changes lie broadly within a 1% range. Hence, if capital is sufficiently immobile across countries, a switch from DP to OP will not put too much pressure on tax revenue.

[Insert Table 5 about here.]

Second, we consider the effect of changing from DP to OP when countries take account of capital mobility in both games. As can be clearly seen from Table 5, when capital is mobile there occurs a very strong race to the bottom: equilibrium tax rates fall tremendously as countries try
to undercut each other to attract industry. Clearly, tax coordination or harmonization becomes of great importance under OP when capital is internationally mobile. While the large difference in the levels of tax rates between the two principles swamps most other considerations, it is worth noting that asymmetries have different effects on tax rates. Whereas under both DP and OP, as countries grow more asymmetric, the larger country increases its tax rate and the smaller country reduces it, the tax gap widens much more under OP than under DP. This larger asymmetry in equilibrium tax rates under OP considerably dampens the home market effect – $\lambda^o$ is much less responsive to increases in $\theta$ than is $\lambda^d$. Hence, when spatial considerations matter (as they surely do in the EU), we cannot dismiss the OP out of hand on the sole basis that it may put pressure on national governments’ tax revenues. We return to this point later in the policy discussion.

Note, finally, that a switch from DP to OP will reduce equilibrium tax rates in both countries, but that the fall will be larger in the small country. This may provide a rationale for why small countries may voice stronger opposition to regime changes than larger countries do.

As can further be seen from Table 5, deeper international integration (lower trade costs $\tau$) has opposite effects on equilibrium tax rates under DP and under OP. Whereas tax rates rise as $\tau$ falls under DP, rates actually fall under OP. The latter effect is straightforward. Indeed, it is well known from the ‘home market effect’ literature that firms’ location choices become more sensitive to market size differences as trade gets freer. Hence, lower values of $\tau$ make the tax base more sensitive to tax differences, thus putting downward pressure on tax rates in both countries as governments undercut each other to increase their share of domestic firms. Under the destination principle, the mechanism works quite differently. Indeed, since the DP tax is a consumption tax, tax revenue does not directly depend on the domestic industry share. As trade gets freer, increased competition leads to lower markups and prices, thereby increasing firms’ demand elasticities and the quantities they sell. Governments may then increase tax rates, because firms price in the more elastic part of their demand and will absorb a substantial part of the tax increase. These findings suggest two important things. First, variable demand elasticity and markups are important when trying to analyze the interactions between trade integration and equilibrium commodity tax rates. Second, trade integration need not necessarily reduce equilibrium tax rates and put more pressure on tax revenue. On the contrary, under DP, trade integration may actually alleviate the problem of tax competition by allowing governments to raise taxes even without any coordinating device.

### 4.4 Tax competition and the spatial distribution of firms

Changes in the tax principle and changes in the nature of tax competition not only affect tax rates and welfare, but also directly influence the degree of spatial inequality in the economy. In a model comparable to ours, Ottaviano and van Ypersele (2005) have shown that there is usually too much agglomeration in equilibrium, so tax differentials provide an instrument to
reduce spatial inequality and increase welfare. As can be seen from Table 1, since equilibrium long run DP tax rates are below both optimal and harmonized rates, the spatial distribution of firms at optimal tax rates is more equal, although the effect seems small in magnitude. Because the long run optimal tax rates under DP result in higher tax rates in both regions, harmonization or cooperation reduce spatial inequalities.

Table 6 compares the long-run spatial allocation under DP to that under OP. As already explained, tax rates fall dramatically with the shift to OP (which strengthens the home market effect), but the larger region also charges the higher tax rates. This tax differential favors firms producing in the smaller region for sales both at home and abroad. Roughly, the switch from DP to OP reduces $\lambda$ by 36% of the gap between $\lambda^d$ and $1/2$. Note that both cooperative and non-cooperative tax setting reduce spatial inequality under OP relative to tax harmonization. This occurs because the tax differential favors the smaller region and is strong enough to attenuate the ‘home market effect’ significantly. Because equilibrium and optimal tax differentials favor the smaller region under OP, harmonization results in more spatial inequality, approximately by the same degree as we find under DP.

4.5 Harmonizing tax rates

Concerns about tax competition as factors become increasingly mobile has led many federations to consider reducing the flexibility of member states in setting their tax rates. There have been EU proposals to place floors under tax rates or to require members to choose tax rates in a band around EU averages. In the USA, states have considerable freedom under the Constitution to choose tax rates and systems, so this has not been a major issue. Within each state, however, local governments obtain all authority to levy taxes and to set tax rates from the state government. State governments have a large variety of policy instruments to prevent tax exporting by local jurisdictions, including setting minimum and maximum rates for particular taxes. An extreme form of preventing tax competition is to take a tax base from local governments and rebate revenue from the tax at a common rate to all local governments in proportion to their shares of that tax base. In practice, it may be quite difficult for a federation to force jurisdictions to levy different tax rates (as opposed to letting countries choose tax rates within a band). It is therefore instructive to consider the issue of complete harmonization where the federation chooses a single

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15In the EU, a minimum VAT rate of 15% has been introduced. In California, the state sales tax includes a 1.25% rate which goes to the local government where the sale occurred. Local government can levy an additional 1.25%, so the mandated portion is only a floor. While the mandated portion may have been an attempt to reduce tax competition, it has led to competition for retail centers through zoning (Lewis and Barbour, 1999).
Tables 1 and 3 present harmonization outcomes under DP and OP in comparison with equilibrium and optimum rates. For DP, the harmonized rate $t^{dh}$ lies between the optimal rates and above both countries’ equilibrium rates. Increasing tax rates and eliminating the gap between them reduce spatial inequalities, although this effect is quantitatively small. The harmonized rate is also quite insensitive to asymmetries in country size. Note also that the welfare losses from harmonization are relatively small under DP, as can be seen from Table 2. It is worth noting that the small country loses and the large country gains relative to the cooperative outcome, which suggests that small countries are less likely to agree to give up their tax-setting powers under DP.

The effects of harmonization under OP are quite different. The harmonized rate lies again between the optimal rates and considerably exceeds the equilibrium rates. As countries become more asymmetric, the harmonized rate increases yet this change is quite small. Most surprisingly, under OP the harmonized rate leads more firms to locate in the larger country – the spatial inequalities are almost as strong as under DP. This effect stems from the fact that in equilibrium the larger country charges higher tax rates under OP than the small country, which reduces firms’ incentive to agglomerate in the large country. Since tax harmonization eliminates this stabilizing mechanism, the spatial inequalities naturally increase in the presence of harmonization.

Along with the concentration of firms in the large country, Table 4 reveals that the small country again loses in a move from the optimum to the harmonized rates, while the large country gains. Thus, under OP, a move to reduce the tax gap between countries leads to greater inequality, both in welfare and spatial terms. While harmonized outcomes are clearly better for both countries than tax competition, the distributional implications may make agreement difficult. As in the DP case, small countries are less likely to agree to give up their tax-setting powers.

5 Policy relevance and conclusions

The questions of which tax principle should be used and whether commodity tax rates should be harmonized are important but contentious issues in several federations like the EU and the USA. Switching from one principle to the other or harmonizing rates is unlikely to leave the economies of member states unaffected, both in the short and in the long run. The most visible short-run impact is that of tax revenue redistribution among the various national and local tax authorities of the federation. It is, for example, estimated that a switch from OP to DP for Texas in-state shipments under the Streamlined Sales Tax Project (SSTP) could “cause a redistribution of local
sales tax revenues from larger urban areas, where goods are purchased to smaller, suburban and rural areas where they are delivered. As much as $160 million in local sales tax revenue could be redistributed in such a manner” (Strayhorn, 2005, p.1). Given the magnitude of the figures, the redistribution of tax revenue attracts the most attention from economists and policy makers. Yet, we have argued that one may miss an important part of the story by focusing exclusively on the direct short-run effects and by neglecting the indirect long-run effects. The latter mainly stem from the redistribution of industry, which may adjust location in order to better exploit tax-differentials across regions. Firms’ locational incentives are themselves largely conditioned by the tax principle applied to commodity transactions. A switch from a ‘consumption based’ tax principle (DP), which falls mainly on immobile consumers, to a ‘production based’ tax principle (OP), which falls mainly on mobile producers, may have important long-run consequences by providing firms with stronger incentives to relocate to low-tax regions. This effect is further amplified by the fact that, in the case of DP, firms established in different locations only differ by the tax rates they face for selling to local consumers; whereas under OP they differ by the tax rate for selling to all consumers in the federation. In our model of tax competition, these two effects of a switch from DP to OP translate into a fierce ‘race to the bottom’, leading to excessively low tax rates and a strong erosion in tax revenues. This result suggests that some form of fiscal harmonization may be desirable to prevent harmful tax competition under OP. On the contrary, welfare losses due to non-cooperative tax-setting under DP are quite low in general, so that harmonization does not appear to be necessary in that case.

Note that the foregoing results are in line with the ones one may expect in most models of tax competition when considering switching the tax burden from the immobile to the mobile agents. Our analysis goes further by highlighting the spatial impacts of such a change in tax principle. This aspect may be quite important for federations with commitment to some regional cohesion objective such as the EU. We have shown that under DP tax competition, by lowering equilibrium tax rates slightly below the optimal tax rates, leads to more spatial inequality, which may interfere with regional cohesion objectives and, therefore, generate additional costs in the form of transfer payments aimed at reducing spatial inequalities.

Concerning the potential impacts of a switch from DP to OP, we have shown that tax revenues decrease significantly whereas the spatial distribution of firms becomes more even because the tax gap between the large and the small country widens. Consequently, although tax revenues decrease, so does spatial inequality. Since there is a tendency for excess agglomeration in equilibrium (Ottaviano and van Ypersele, 2005), less agglomeration may actually be welfare improving as fewer resources, such as the structural funds of the EU, are required to alleviate regional inequalities. How tax revenue and spatial inequality are traded off is, ultimately, a political and societal question of the ‘efficiency vs. equity’ type to which our model can provide no answer. The gains from switching to OP are lower costs of collecting and remitting tax revenues, as well as less spatial inequality, whereas the costs are lower tax revenues. Although this trade off can
only be resolved by knowing society’s ‘aversion for spatial inequality’, which we have not specified in our model, it is important to point out that given the magnitudes of welfare differences under OP and DP (see Table 6 for a summary of the figures), it is unlikely that forcing a more equal spatial distribution under OP offsets the high costs in terms of foregone tax revenue. From an EU perspective, a switch to an origin-based commodity taxation may therefore put strain on the integration process and undermine the financing of the welfare state, triggering strong resistance especially from the large contributing regions.\textsuperscript{17}

Note, finally, that DP equilibrium rates increase as international integration proceeds, whereas OP rates clearly fall. This unsuspected result shows that more integration need not erode tax revenue under DP, thereby suggesting that DP may remain a preferable option in an increasingly integrated economy.

To conclude, we find that there is no general presumption in favor of OP based commodity taxation. Neither is there in favor of DP based commodity taxation. Both systems display strong and weak features which make them ultimately difficult to compare. This finding may explain why proposals to reform commodity taxation, as advocated by some members in the EU and in the USA, have not generally met broad agreement and quick implementation of changes by the other members.

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**References**


\textsuperscript{17}As in Haufler and Pflüger (2004), we find that DP may actually dominate OP, provided the federation does not care too much about spatial inequalities. However, contrary to Haufler and Pflüger (2004), where all equilibrium taxes are negative and inefficiently high, the non-cooperative tax rates are positive and inefficiently low in our model.


Appendix

A.1. Trade conditions: For international demands $q_{HF}$ and $q_{FH}$ to remain strictly positive for all firm distributions $0 < \lambda < 1$ under both tax principles, it must be that:

$$\tau < \frac{2a}{2b+c} - m \frac{2b}{2b+c} \max \{s^d_H, s^d_F\}$$

under DP, and

$$\tau < \frac{2a}{2b+c} - \max_{i \neq j} \frac{2b}{2b+c} \left[ \frac{2b}{2b+c} \left(1 + s^o_i - (s^o_j - s^o_i) \right) \right]$$

under OP. We assume throughout the paper that (11) and (12) hold.
A.2. Spatial equilibrium under DP: The unique solution to the equilibrium condition $r^d_H = r^d_F$ is given by:

$$
\chi^d = \frac{1}{2} + \frac{2}{c^\tau} \left[ \frac{2a - b\tau - 2bm(1 + s^d_H)(1 + s^d_F)}{1 + \theta s^d_F + (1 - \theta)s^d_H} \right] \left( \theta - \frac{1}{2} \right)
+ \frac{2a - b\tau}{c^\tau} \left[ \theta s^d_F - (1 - \theta)s^d_H \right].
$$

A.3. Price derivatives: The price derivatives are simple under the destination principle:

$$
\frac{\partial p^d_{FH}}{\partial s^d_H} = \frac{\partial p^d_{FH}}{\partial s^d_F} = \frac{\partial p^d_{HF}}{\partial s^d_F} = \frac{\partial p^d_{HF}}{\partial s^d_F} = \frac{b + c}{2b + c} m > 0 \quad \text{and} \quad \frac{\partial p^d_{FH}}{\partial s^d_H} = \frac{\partial p^d_{HF}}{\partial s^d_H} = \frac{\partial p^d_{HF}}{\partial s^d_H} = \frac{\partial p^d_{HF}}{\partial s^d_H} = 0.
$$

Under the origin principle, we need to distinguish between firms established in country $H$ and firms established in country $F$. In country $F$, the derivatives are given by:

$$
\frac{\partial p^o_{FH}}{\partial s^o_H} = \frac{b + c}{2b + c} m - \frac{c(1 - \lambda)m}{2(2b + c)} > 0 \quad \text{and} \quad \frac{\partial p^o_{FH}}{\partial s^o_F} = \frac{c(1 - \lambda)m}{2(2b + c)} > 0
$$

whereas for firms established in country $F$ are as follows:

$$
\frac{\partial p^d_{FH}}{\partial s^d_F} = \frac{b + c}{2b + c} m - \frac{c\lambda m}{2(2b + c)} > 0 \quad \text{and} \quad \frac{\partial p^d_{FH}}{\partial s^d_H} = \frac{c\lambda m}{2(2b + c)} > 0.
$$

B. Numerical implementation: Under OP, the rental rates of capital is given by (7). All $\lambda$ terms are buried in the prices and enter those terms linearly. The spatial equilibrium is a solution to the quadratic equation:

$$
\Delta (\lambda; s^o_H, s^o_F) = r^o_H (\lambda; s^o_H) - r^o_F (\lambda; s^o_F) = 0.
$$

For a stable equilibrium, if $\lambda > \lambda^*$, then $\Delta (\lambda; s^o_H, s^o_F) < 0$. Thus, a stable equilibrium has $d\Delta/d\lambda < 0$. Given that $\Delta$ is a quadratic equation in $\lambda$, we can write it as:

$$
\Delta (\lambda; s^o_H, s^o_F) = A\lambda^2 + B\lambda + C = 0.
$$

Since $d\Delta/d\lambda = 2A\lambda + B$, the sign of $A$ determines which root is the stable one (negative derivative with respect to $\lambda$ in the neighborhood of the root). If $A > 0$, the smaller root (the one subtracting the discriminant) is stable, while if $A < 0$, the larger root is the stable one. Writing $\lambda^* = -B/2A \pm \sqrt{B^2 - 4AC}/2A$, the stable root always subtracts the discriminant. Note, finally, that when $s^o_F = s^o_H$, the equation is linear in $\lambda$. Since $B < 0$, the linear equation solution is always stable. An easier approach that we can use if there is only one solution with $0 < \lambda < 1$ (which is the case for our benchmark set of parameter values) is to take $\Delta (\lambda; s^o_H, s^o_F) = r^o_H (\lambda; s^o_H) - r^o_F (\lambda; s^o_F) = 0$ and view it as a (well-behaved) level curve and totally differentiate to find the expressions

$$
\frac{\partial \lambda}{\partial s^o_H} \bigg|_{\Delta=0} \quad \text{and} \quad \frac{\partial \lambda}{\partial s^o_F} \bigg|_{\Delta=0}.
$$
We then simply have

$$\left. \frac{\partial \lambda}{\partial s_H^o} \right|_{\Delta=0} = - \left( \frac{\partial r_H^o}{\partial s_H^o} - \frac{\partial r_F^o}{\partial s_H^o} \right) \left( \frac{\partial r_H^o}{\partial \lambda} - \frac{\partial r_F^o}{\partial \lambda} \right)^{-1}$$

Making the change of variable

$$\tilde{\lambda} \equiv \ln \left( \frac{\lambda}{1 - \lambda} \right)$$

and searching over $\tilde{\lambda}$, we can evaluate the derivative of $\lambda$ with respect to $s_H^o$ and $s_F^o$ numerically in a straightforward way. We implemented this procedure using Matlab and the associated optimization toolbox. The programs are available from the authors upon request.
### Table 1 — Equilibrium, cooperative, and harmonized rates under long-run DP

<table>
<thead>
<tr>
<th>θ</th>
<th>$t^d_H$</th>
<th>$t^d_F$</th>
<th>$\lambda^d$</th>
<th>$t^d_H$</th>
<th>$t^d_F$</th>
<th>$\lambda^d$</th>
<th>$t^d_H$</th>
<th>$t^d_F$</th>
<th>$\lambda^d$</th>
<th>$t^d_H$</th>
<th>$t^d_F$</th>
<th>$\lambda^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.1053</td>
<td>0.1053</td>
<td>0.5000</td>
<td>0.1248</td>
<td>0.1248</td>
<td>0.5000</td>
<td>0.1248</td>
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<td></td>
</tr>
<tr>
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<td>0.5236</td>
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</tr>
<tr>
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<td>0.5482</td>
<td>0.1250</td>
<td>0.1246</td>
<td>0.5454</td>
<td>0.1248</td>
<td>0.5472</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.53</td>
<td>0.1061</td>
<td>0.1046</td>
<td>0.5723</td>
<td>0.1251</td>
<td>0.1244</td>
<td>0.5680</td>
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<td>0.5708</td>
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<td></td>
<td></td>
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<tr>
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<td>0.1041</td>
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<td>0.6180</td>
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<td></td>
</tr>
</tbody>
</table>

Notes: $m = 0.7$, $\beta = 0.25$, $\gamma = 0.2$, $\alpha = 1$ and $\tau = 0.07$.

### Table 2 — Long-run DP welfare changes with respect to the cooperative case

<table>
<thead>
<tr>
<th>θ</th>
<th>$W^{dc}_H$</th>
<th>$W^{dc}_F$</th>
<th>Welfare changes when switching from unified DP to Long-run DP</th>
<th>$W^{dh}_H$</th>
<th>$W^{dh}_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5000</td>
<td>0.6995</td>
<td>0.6995</td>
<td>-0.20</td>
<td>-0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5100</td>
<td>0.7147</td>
<td>0.6843</td>
<td>-0.21</td>
<td>-0.22</td>
<td>0.01</td>
</tr>
<tr>
<td>0.5200</td>
<td>0.7300</td>
<td>0.6692</td>
<td>-0.22</td>
<td>-0.22</td>
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<tr>
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<td>-0.23</td>
<td>0.02</td>
</tr>
<tr>
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<td>-0.24</td>
<td>-0.24</td>
<td>0.02</td>
</tr>
<tr>
<td>0.5500</td>
<td>0.7759</td>
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<td>-0.25</td>
<td>-0.25</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Notes: $m = 0.7$, $\beta = 0.25$, $\gamma = 0.2$, $\alpha = 1$ and $\tau = 0.07$.

### Table 3 — Equilibrium, cooperative, and harmonized rates under long-run OP

<table>
<thead>
<tr>
<th>θ</th>
<th>$t^{o*}_H$</th>
<th>$t^{o*}_F$</th>
<th>$\lambda^{o*}$</th>
<th>$t^{oc}_H$</th>
<th>$t^{oc}_F$</th>
<th>$\lambda^{ou}$</th>
<th>$t^{oh}$</th>
<th>$\lambda^{oh}$</th>
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</thead>
<tbody>
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<td>0.0154</td>
<td>0.5000</td>
<td>0.1248</td>
<td>0.1248</td>
<td>0.5000</td>
<td>0.1248</td>
<td>0.5000</td>
</tr>
<tr>
<td>0.51</td>
<td>0.0159</td>
<td>0.0148</td>
<td>0.5153</td>
<td>0.1252</td>
<td>0.1244</td>
<td>0.5103</td>
<td>0.1249</td>
<td>0.5236</td>
</tr>
<tr>
<td>0.52</td>
<td>0.0163</td>
<td>0.0142</td>
<td>0.5306</td>
<td>0.1256</td>
<td>0.1240</td>
<td>0.5206</td>
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<td>0.5472</td>
</tr>
<tr>
<td>0.53</td>
<td>0.0168</td>
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<td>0.5461</td>
<td>0.1260</td>
<td>0.1235</td>
<td>0.5309</td>
<td>0.1251</td>
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<td>0.0172</td>
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</tr>
</tbody>
</table>

Notes: $m = 0.7$, $\beta = 0.25$, $\gamma = 0.2$, $\alpha = 1$ and $\tau = 0.07$.  

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Table 4 — Long-run OP welfare changes with respect to the cooperative case

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Cooperative OP</th>
<th>Welfare changes when switching from unified OP to Harmonized OP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W^o_H$</td>
<td>$W^o_F$</td>
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<tr>
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<tr>
<td>0.5200</td>
<td>0.7295</td>
<td>0.6696</td>
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<td>0.5300</td>
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<tr>
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</table>

Notes: $m = 0.7, \beta = 0.25, \gamma = 0.2, \alpha = 1$ and $\tau = 0.07.$

Table 5 — Switch from long-run DP to either short- or long-run OP

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\tau$</th>
<th>Long-run DP</th>
<th>Short-run OP</th>
<th>Long-run OP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$t^d_H$</td>
<td>$t^d_F$</td>
<td>$\lambda^d$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.05</td>
<td>0.1068</td>
<td>0.1068</td>
<td>0.5000</td>
</tr>
<tr>
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</tr>
<tr>
<td>0.52</td>
<td>0.05</td>
<td>0.1073</td>
<td>0.1062</td>
<td>0.5707</td>
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<tr>
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<td>0.1076</td>
<td>0.1060</td>
<td>0.6061</td>
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<tr>
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<td>0.1079</td>
<td>0.1057</td>
<td>0.6414</td>
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<tr>
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<td>0.1055</td>
<td>0.6768</td>
</tr>
<tr>
<td>0.5</td>
<td>0.07</td>
<td>0.1053</td>
<td>0.1053</td>
<td>0.5000</td>
</tr>
<tr>
<td>0.51</td>
<td>0.07</td>
<td>0.1056</td>
<td>0.1050</td>
<td>0.5241</td>
</tr>
<tr>
<td>0.52</td>
<td>0.07</td>
<td>0.1058</td>
<td>0.1048</td>
<td>0.5482</td>
</tr>
<tr>
<td>0.53</td>
<td>0.07</td>
<td>0.1061</td>
<td>0.1046</td>
<td>0.5723</td>
</tr>
<tr>
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<td>0.07</td>
<td>0.1064</td>
<td>0.1043</td>
<td>0.5964</td>
</tr>
<tr>
<td>0.55</td>
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<td>0.1067</td>
<td>0.1041</td>
<td>0.6204</td>
</tr>
</tbody>
</table>

Notes: $m = 0.7, \beta = 0.25, \gamma = 0.2, \alpha = 1$.

Table 6 — Changes in welfare and industry location under long-run DP and long-run OP

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Long-run DP and OP</th>
<th>$\Delta$ W DP to OP in %</th>
<th>Industry location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>$W^{ds}_F$</td>
<td>$W^{os}_H$</td>
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<td>0.6981</td>
<td>0.6375</td>
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<tr>
<td>0.5100</td>
<td>0.7134</td>
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<td>0.6522</td>
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<tr>
<td>0.5200</td>
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<td>0.6669</td>
</tr>
<tr>
<td>0.5300</td>
<td>0.7439</td>
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<td>0.6817</td>
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<tr>
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<td>0.6966</td>
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</table>

Notes: $m = 0.7, \beta = 0.25, \gamma = 0.2, \alpha = 1$ and $\tau = 0.07.$