Limited Asset Market Participation: Does it Really Matter for Monetary Policy?

Guido Ascari  Andrea Colciago
University of Pavia and Kiel IfW  University of Milano-Bicocca
Lorenza Rossi
University of Pavia

PRELIMINARY - PLEASE DO NOT CIRCULATE

Abstract

We study Ramsey policies and optimal monetary policy rules in a model with sticky wages and prices, where a fraction of consumer are liquidity constrained. The interaction between liquidity coinstrained agents and wage stickiness results in a welfare-based loss function which depends on real wage gap beside on the output gap, wage inflation and price inflation. Nevertheless, contrary to previous findings: i) LAMP does not matter much for the dynamics of a NK model; 2) optimal simple rules are characterized by active monetary policies, no matter the degree of LAMP. We argue that once wage stickiness, an uncontroversial empirical fact, is considered, the degree of financial market participation do not affect the design of optimal monetary policy rules.

JEL Classification Numbers: E0, E4, E5, E6.

Keywords: optimal monetary policy, sticky wages, liquidity constrained household, determinacy, optimal simple rules.
1 Introduction

In this paper, we study optimal monetary policy in a New Keynesian (NK henceforth) framework characterized by nominal price and wage stickiness together with two types of households: (i) households who cannot trade on financial markets and therefore consume all their current disposable income, labelled as liquidity constrained consumers; (ii) standard Ricardian households who have access to a full set of state contingent securities and therefore are able to smooth consumption.\(^1\) In particular, building on Galí et al (2004, 2007) we provide a model which nests two popular environments in the optimal monetary policy literature. The first one is that suggested by Erceg et al. (2000), which is a fully Ricardian NK model characterized by price and wage stickiness. The second one is the Limited Asset Market Participation (LAMP henceforth) model of Bilbiie (2008), which is characterized by a positive fraction of liquidity constrained consumers, but by flexible wages. Both the assumptions of nominal wage stickiness and LAMP are of particular interest, at least for two reasons: 1) nominal wage stickiness and LAMP represent two important stylized facts;\(^2\) 2) optimal monetary policy prescriptions seem to be dramatically altered when either wage stickiness or LAMP is introduced in the basic NK model. In particular, Erceg at al (2000) show that, with nominal wage stickiness, households' unconditional welfare depends not only on the unconditional variances of output gap and price inflation, but also on wage inflation. The central bank faces a trade-off in stabilizing the output gap, price inflation, and wage inflation. Consequently, strict price inflation targeting generates relatively large welfare losses, whereas several other simple policy rules, which target wage inflation or output gap together with price inflation perform nearly as well as the optimal rule.

Bilbiie (2008) shows that, LAMP substantially alters the dynamics of a standard

\(^{1}\)In a recent paper Mankiw (2000) argues that the behaviour of consumer which do not smooth consumption can be viewed as resulting from i) myopic deviations from the assumption of fully rational expectations (rule-of-thumb consumers); ii) consumers who face binding borrowing constraints (liquidity constrained consumers).

\(^{2}\)Christiano et al (2005) and Smets and Wouters (2003) for examples show that nominal rigidity are important sources of monetary non-neutralities.

Regardless of whether aggregate time series or micro data are used, consumption tracks current income far more closely than suggested by the Permanent Income Hypothesis. For example, Vissing-Jorgensen (2002) reports based on the PSID data that of US population 21.75% hold stock and 31.40% hold bonds. A large part of population holds no interest bearing asset and has no checking account either
NK model by causing a change in the sign of the slope of the IS-curve and inverting the Taylor principle. Therefore, Bilbiie (2008) argues that in the pre-Volcker period the economy was in what he called the inverted aggregate demand logic (IADL henceforth) region, and then it would have been optimal for the Fed to implement a passive policy.

The aim of this paper is to study how the two assumptions of LAMP and nominal wage stickiness interact, affect the dynamics and the stability properties of a NK model as well as the prescriptions for the optimal monetary policy. In particular, we derive the society’s welfare-loss function as a second order approximation of a weighted average of the two representative households’ lifetime utilities. Then, we study Ramsey monetary policy under commitment and the optimal monetary policy rules. We find the following results. First, the interaction between LAMP and wage stickiness results in a welfare-based loss function which depends on real wage gap beside on the output gap, wage inflation and price inflation. Second, the elasticity of the aggregate demand with respect to the real interest rate increases as nominal wage stickiness increases. Third, the inversion of the slope of the IS occurs only for very high value of LAMP. A direct consequence of the former result is that, liquidity constrained consumers substantially do not matter to define the determinacy region for monetary policy rules. Our main finding however is that on optimal monetary policy. We show that, contrary to what found by Bilbiie (2008), LAMP does not have any relevant implications for the design of optimal monetary policy, once wage stickiness is taken into account. In other words, the results obtained by Erceg et al (2000) under nominal wage stickiness are robust to the introduction of LAMP, while the results obtained by Bilbiie (2008) with LAMP are not robust to the introduction of nominal wage stickiness. Indeed, we find that, optimal simple rules are always characterized by active monetary policies, no matter the degree of financial market participation. Furthermore, our model calls for a stronger reaction to inflation as the share of liquidity constrained agents increases. A conclusion which does not support, the interpretation of the great inflation given in Bilbiie (2008), who claims that the higher inflation volatility of the pre-Volcker period could simply be due to the optimal behavior of the Federal Reserve.

This paper is related to several other works analyzing the role of LAMP in NK models. In particular, as in Bilbiie (2008), most of the papers which focus on monetary policy and determinacy analysis in NK models with LAMP, consider a labor market characterized by flexible wages. For examples, Amato and Laubach (2003) by modeling
consumers’ rule-of-thumb behavior as a consumption habit, show that the optimal interest rate becomes much more intertial as the fraction of liquidity constrained consumers increases. Di Bartolomeo and Rossi (2007), show that monetary policy effectiveness increases as the degree of LAMP increases. Galì et al. (2004), consider a NK model with capital accumulation and show that the Taylor principle may become a too weak criterion for stability when LAMP increases. Finally, Di Bartolomeo et al. (2010) introduce external habits in consumption. By using Bayesian methods to estimates the model structural parameters of the G7 countries, they show that even with a fraction of LAMP significant in many countries (0.26 on average), in none of them this fraction is high enough to generate the regime inversion obtained by Bilbiie (2008).

The paper of Galì et al. (2007), is the first attempt to study fiscal policy in a NK model with LAMP, but still with flexible wages. The authors show that the interaction of LAMP with sticky prices and deficit financing can account for the existing evidence on consumption crowding-in following an increase in government expenditure.

While all the models afore-mentioned, consider a labor market characterized by fully flexible nominal wages, the few papers which embeds nominal wage stickiness in a model with LAMP focus on fiscal policy analysis. For example, Coenen and Straub (2005) by estimating a NK model of the euro area, find a small chance that government spending shocks crowd in consumption, mainly because the estimates of the values of LAMP are relatively low. Colciago (2008) finds that with nominal wage stickiness: (i) the standard results on the determinacy of the equilibrium of the NK model are almost restored; (ii) results obtained by Galì et al. (2007) on the effects of Government spending shocks on consumption are not robust and hold just in a very particular case. Finally, Forni et al. (2009) build a medium-scale NK model with LAMP and estimate the effects of fiscal policy in the Euro area. They find a share of non-Ricardian agents of about 40% and only mild Keynesian effects of fiscal policy.

None of the works quoted analyses optimal monetary policy in a NK model with sticky wages and LAMP and, to the best of our knowledge our model is the first attempt to fill this gap. Summing up, we find some new results, and in particular: 1) we extend the results obtained by Colciago (2008) on the determinacy properties of a NK with LAMP; 2) we show that contrary to what is stated in Bilbiie (2008), LAMP has negligible and only quantitative effects on the conduct of the optimal monetary policy.

Indeed, he shows that the IALD region survives only for very high value of LAMP.
The estimates on the fraction of liquidity consumers obtained in some recent empirical works (Coenen and Straub 2005, Di Bartolomeo et al 2010 and Forni et al 2009), strongly support our statement that LAMP does not matter for monetary policy once nominal wage stickiness, an incontrovertible empirical fact, is considered.

The paper is organized as follows. Section 2 describes the model and section 3 studies its aggregate dynamics and the determinacy properties. Section 4 derives the welfare-loss of the central bank as a second order approximation of the society's utility function and derives the optimal monetary policy under commitment. Section 5 looks for the optimal, simple and implementable monetary policy rules. Section 6 concludes.

2 The Model


2.1 Households

The economy features a continuum of households indexed by $i \in [0, 1]$. Households in the interval $[0, \lambda]$ cannot access financial markets, and we label them as liquidity constrained consumers or non-Ricardian households. They consume their available labor income in each period. The rest of the households on the interval $(\lambda, 1]$ is composed by standard Ricardian households who have access to a full set of state contingent securities. The period utility function is common across households and it has the following separable form:

$$U_t = \Psi_t u [C_t (i)] - v [L_t (i)],$$

where $C_t (i)$ is agent $i$'s consumption and $L_t (i)$ are hours worked. $u$ and $v$ satisfy the usual properties, $^4$ and $\Psi_t$ is a taste shock. A positive change in $\Psi_t$ delivers an increase in marginal utility of consumption and leads to an urgency to consume. The modelling of the labor market follows Colciago (2006). We assume a continuum of differentiated labor inputs indexed by $j \in [0, 1]$, and corresponding labor type-specific unions. Given the wage $W^j_t$ fixed by union $j$, agents supply the hours on labor market of labor type $j$.

$^4$The function $u$ is increasing and concave and the function $v$ is increasing and convex.
$L_t^j$, demanded by firms (see below), that is:

$$L_t^j = \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} L_t^d,$$

where $\theta_w$ is the elasticity of substitution between labor inputs. Here $L_t^d$ is aggregate labor demand and $W_t$ is an index of the wages prevailing in the economy at time $t$. Formal definitions of labor demand and of the wage index can be found in the section devoted to firms. Agents are distributed uniformly across unions, hence aggregate demand of labor type $j$ is spread uniformly between all households.\textsuperscript{5} It follows that the individual quantity of hours worked, $L_t(i)$, is common across households and we will denote it with $L_t$. This must satisfy the time resource constraint $L_t = \int_0^1 L_t^j dj$. Combining the latter with (2) we obtain:

$$L_t = L_t^d \int_0^1 \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} dj.$$

The labor market structure allows to rule out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by $L_t^d \int_0^1 W_t^j \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} dj$.\textsuperscript{6}

\textbf{2.1.1 Ricardian Households}

Ricardian households face the following flow budget constraint (where we distinguish shares from the other assets explicitly since their distribution plays a crucial role in the rest of the analysis):

$$E_t \Lambda_{t+1} X_{t+1} + \Omega_{S,t+1} V_t \leq X_t + L_t^d \int_0^1 W_t^j \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} dj + \Omega_{S,t} (V_t + P_t D_t) - P_t C_{S,t}.$$

\textsuperscript{5}Thus a share $\lambda$ of the associates of the unions are non ricardian consumers, while the remaining share is composed by non ricardian agents.

\textsuperscript{6}Our assumption is similar to Woodford (2003) among others, but different from the one in Erceg et al. (2000). As in most of the literature on sticky wages, Erceg et al. (2000) assume that each agent is the monopolistic supplier of a single labor input. In this case, only households providing the same labor type will exhibit the same labor income. However, the assumption of complete markets and full insurance against the risk associated to labor income fluctuations, rule out differences in income between households. In our model, however, this framework would imply a tractability problem, because non ricardian agents do not participate in the asset market, and thus cannot share the risk associated to labor income fluctuations.
In each period $t$, ricardian agents (indicated with the subscript $S$) can purchase any desired state-contingent nominal payment $X_{t+1}$ in period $t+1$ at the dollar cost $E_t \Lambda_{t,t+1} X_{t+1}$. The variable $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t+1$ and $t$. The expression $L_t^d \int_0^1 W_t^j \left( \frac{W_t^j}{W_t^j} \right)^{-\theta_w} dj$ represents labor income. $V_t$ is average market value at time $t$ of the shares in intermediate good firms, $D_t$ are real dividend payoffs of these shares and $\Omega_{s,t}$ are share holdings. FOCS with respect to $C_{S,t}$, $\Omega_{S,t}$ and $X_{t+1}$ are respectively:

$$\Psi_t u_c(C_{S,t}) = v_t P_t,$$  \hspace{1cm} (5)
$$V_t = E_t \beta \frac{V_{t+1}}{v_t} (V_{t+1} + P_{t+1} D_{t+1}),$$  \hspace{1cm} (6)
$$\Lambda_{t,t+1} = \beta \frac{V_{t+1}}{v_t},$$  \hspace{1cm} (7)

where $v_t$ is the Lagrange multiplier on the flow budget constraint. Since $\Lambda_{t,t+1} \equiv (1 + i_t)^{-1}$, the Euler equation for ricardian agents is:

$$\frac{1}{1 + i_t} = E_t \left\{ \beta \frac{\Psi_{t+1} u_c(C_{S,t+1})}{\Psi_t u_c(C_{S,t})} \frac{P_t}{P_{t+1}} \right\}.$$  \hspace{1cm} (8)

### 2.1.2 Liquidity Constrained Households

Liquidity constrained agents (indicated with the subscript $H$) do not enjoy firms’ profits in the form of dividend income and cannot trade in the financial markets. The nominal budget constraint of a typical liquidity constrained household is thus simply given by:

$$P_t C_{H,t} = L_t^d \int_0^1 W_t^j \left( \frac{W_t^j}{W_t^j} \right)^{-\theta_w} dj.$$  \hspace{1cm} (9)

This type of agents are forced to consume disposable income in each period and delegate wage decisions to unions. For these reasons there are no first order conditions with respect to consumption and labor supply.

### 2.2 Wage Setting

Nominal wage rigidities are modeled according to the Calvo (1983) mechanism. In each period a union faces a constant probability $1 - \xi_w$ of being able to re-optimize the nominal wage. We extend the analysis in Galì et al. (2007) and assume that the nominal wage newly reset at $t$, $\tilde{W}_t$, is chosen to maximize a weighted average of agents’ lifetime


utilities. The weights attached to the utilities of ricardian and liquidity constrained agents are \((1 - \lambda)\) and \(\lambda\), respectively. The union problem is then:

\[
\max_{W_t} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \xi_w)^k \{ \Psi_{t+k} [(1 - \lambda) u (C_{S,t+k}) + \lambda u (C_{H,t+k})] - v (L_{t+k}) \},
\]

subject to (4) and (9). The FOC with respect to \(W_t\) is:

\[
\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \xi_w)^k \Phi_{t,t+k} \left\{ \left( 1 - \lambda \right) \frac{1}{MRS_{S,t+k}} + \lambda \frac{1}{MRS_{H,t+k}} \right\} \tilde{W}_t \left\{ - (1 + w^*) \right\} = 0,
\]

where \(\Phi_{t,t+k} = v_L (L_{t+k}) L_{t+k}^{\theta_w} W_{t+k}^{\theta_w} \) and \(w^* = (\theta_w - 1)^{-1}\) is the constant net wage mark-up in the case of wage flexibility. The variables \(MRS_{H,t}\) and \(MRS_{S,t}\) denote the marginal rates of substitution between labor and consumption of liquidity constrained and ricardian agents respectively.

2.3 Firms

In each period \(t\), a final good \(Y_t\) is produced by perfectly competitive firms combining a continuum of intermediate inputs \(Y_t (z)\) according to the following standard CES production function:

\[
Y_t = \left( \int_0^1 Y_t (z) \frac{\theta_p^{-1}}{\theta_p} dz \right)^{\frac{\theta_p}{\theta_p - 1}} \text{ with } \theta_p > 1
\]

The competitive final goods producers’ demand of the intermediate good \(z\) and the price of the final good are thus equal to:

\[
Y_t (z) = \left( \frac{P_t (z)}{Y_t} \right)^{-\theta_p} Y_t \quad \text{;} \quad P_t = \left[ \int_0^1 P_t (z)^{1-\theta_p} dz \right]^{\frac{1}{1-\theta_p}}.
\]

Intermediate inputs are produced by a continuum of monopolistic firms indexed by \(z \in [0, 1]\). The production technology is simply linear in labor services, \(L_t (z)\):

\[
Y_t (z) = A_t L_t (z),
\]

where \(A_t\) represents TFP. The labor input is defined as \(L_t (z) = \left( \int_0^1 \left( L_t^j (z) \right)^{\frac{\theta_w-1}{\theta_w}} dz \right)^{\frac{\theta_w}{\theta_w-1}} \).

Firm’s \(z\) demand for labor type \(j\) and the aggregate wage index are then respectively:

\(^7\)Many reasons have been provided to justify the presence of non ricardian consumers. A few of them are miopia, fear of saving and transaction costs on financial markets. None of these is, however, in contrast with rule of thumb consumers delegating wage decision to a forward looking agency, in this case a trade union.
\[ L_t^j(z) = \left( \frac{W_t^j}{W_t^j} \right)^{-\theta_w} L_t(z) ; \quad W_t = \left( \int_0^1 \left( W_t^j \right)^{1-\theta_w} dj \right)^{1/(1-\theta_w)}. \]

Note that the nominal marginal cost is common across producers and it is given by
\[ MC_t = \frac{W_t}{A_t}. \quad (14) \]

Finally, firm z’s real profits are
\[ D_t(z) = \left[ \frac{P_t(z)}{P_t} - \frac{MC_t}{P_t} \right] Y_t(z). \quad (15) \]

### 2.4 Price Setting

Intermediate producers set prices according to the same mechanism assumed for wage setting. Firms in each period have a fixed chance \( 1 - \xi_p \) to re-optimize their price. A price setter takes into account that the choice of its time \( t \) nominal price, \( P_t \), might affect not only current but also future profits. The FOC for price setting is:
\[ E_t \sum_{k=0}^{\infty} (\beta \xi_p)^k v_{t+k} P_t^{\rho_p} Y_{t+k} \left[ \frac{P_t}{P_t} - (1 + \mu^p) MC_{t+k} \right] = 0, \quad (16) \]

which can be given the usual interpretation.\(^8\) Notice that \( \mu^p = (\theta_p - 1)^{-1} \) represents the net markup over the price which would prevail in the absence of nominal rigidities.

### 2.5 Aggregation and Market Clearing

Aggregate consumption is given by:
\[ C_t = \lambda C_{H,t} + (1 - \lambda) C_{S,t}. \quad (17) \]

Since each ricardian agents holds the same quantity of shares and the sum of shares must equal 1, each agents hold a fraction \( (1 - \lambda) \) of total shares:
\[ \Omega_t = (1 - \lambda) \Omega_{S,t} \Rightarrow \Omega_{S,t} = \Omega_S = \frac{1}{1 - \lambda}. \quad (18) \]

The clearing of good and labor markets requires
\[ \begin{cases} Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\theta_p} Y_t^d, \quad \forall z, \quad Y_t^d = Y_t; \\ L_t^j = \left( \frac{W_t^j}{W_t} \right)^{-\theta_w} L_t^d, \quad \forall j, \quad L_t = \int_0^1 L_t^j dj, \end{cases} \quad (19) \]

\(^8\)Recall that \( v_t \) is the lagrange multiplier on ricardian households nominal flow budget constraint. \( v_t \) is thus the value of an additional dollar for ricardian households, who own the firm shares.
where $Y^d_t = C_t$ represents aggregate demand, $L^j_t = \int_0^1 L^j_t(z) \, dz$ is total aggregate demand of labor input $j$ and $L^d_t = \int_0^1 L_t(z) \, dz$ denotes firms’ aggregate demand of the composite labor input $L_t$.

### 2.6 The Steady State

As in Bilbiie (2008) society’s welfare loss will be represented by a second order approximation to a weighted average of households lifetime utilities, where weights are given by the relative importance of agents’s groups in the economy. In order to study the welfare properties of the economy without resorting to a full second order approximation to the model equations, we assume an efficient steady state of the economy. More precisely, as usual in this literature (see Woodford, 2003), we assume that the government taxes firms by means of a constant employment tax, $\tau$, at the steady state, and then gives the money back to firms through lump-sum transfers, denoted with $T$. In this case the typical firm’s profit function at the steady state reads as:

$$D = \frac{P}{P} Y - \frac{(1 - \tau) W}{P} L - T,$$

where to balance the government budget we assume that $\tau \frac{W}{P} L = T$. The efficient steady state must be characterized by zero profits, and by equal consumption by the two types of agents: $C_S = C_L = C$. It follows that agents have a common marginal rate of substitution between labor and consumption, denoted by $MRS$. In this case the steady state labor market equilibrium implies:

$$\frac{W}{P} = \frac{1}{(1 + \mu_p)(1 - \tau)} MPL = (1 + \mu^w) MRS.$$

(21)

Given the selected production function, at the efficient steady state it has to be the case that

$$MPL = MRS = 1,$$

(22)

where $MPL$ is the marginal product of labor. From equation (21), the latter condition is satisfied if

$$\tau = 1 - \frac{1}{(1 + \mu_p)(1 + \mu^w)}$$

(23)

As argued above the implied value of $\tau$ leads to zero steady state profits

$$D = Y - \frac{(1 - \tau) W}{P} L - T = Y - \frac{Y}{(1 + \mu_p)^2 (1 + \mu^w)} - \left( 1 - \frac{1}{(1 + \mu_p)^2 (1 + \mu^w)} \right) Y = 0$$
2.7 Flexible Prices Equilibrium

In this section we define the equilibrium of the model under flexible prices and wages. Since we remove the steady state distortions, we will refer to this equilibrium as to the efficient equilibrium. Appendix A.1 shows that the log-deviations from the efficient steady state of the efficient output, the efficient real wage and the efficient real rate of interest are respectively given by:\footnote{We denote log-deviations by lower case letters, and $\omega$ stands for the log-deviation of the real wage.}

\begin{align}
y_{t}^{Eff} &= \frac{1 + \phi}{\sigma + \phi} a_t + \frac{1}{(\sigma + \phi)} \psi_t, \tag{24} \\
\omega_{t}^{Eff} &= a_t, \tag{25} \\
r_{t}^{Eff} &= \sigma \left( \frac{1 + \phi}{\sigma + \phi} \Delta a_{t+1} - \frac{\phi}{\sigma (\sigma + \phi)} \Delta \psi_{t+1} \right). \tag{26}
\end{align}

Assuming an AR(1) process for the logarithms of the exogenous state variables:

\begin{align}
a_t &= \rho^a a_{t-1} + \varepsilon^a_t \tag{27} \\
\psi_t &= \rho^\psi a_{t-1} + \varepsilon^\psi_t \tag{28}
\end{align}

fully specifies the dynamics of the log-deviations form the efficient equilibrium.

2.8 The Log-linear model

The following equations describe the log-linearized model:

\begin{align}
(M1) \quad \pi_t &= \beta E_t \pi_{t+1} + \kappa_{\pi} \tilde{\omega}_t & \text{New Keynesian Phillips Curve (NKPC)} \\
(M2) \quad \pi_w^t &= \beta E_t \pi_{t+1}^w + \kappa_w [(\sigma + \phi) x_t - \tilde{\omega}_t] & \text{Wage Inflation Curve} \\
(M3) \quad \tilde{\omega}_t &= \tilde{\omega}_{t-1} + \pi_w^t - \pi_t + \Delta \omega_t^{Eff} & \text{Real Wage Gap} \\
(M4) \quad x_t &= E_t x_{t+1} - \frac{1}{\sigma} E_t \left( i_t - \pi_{t+1} - r_t^{Eff} \right) - \frac{\lambda}{1-\lambda} E_t \Delta \tilde{\omega}_{t+1} & \text{IS curve}
\end{align}

The first equation is the celebrated NKPC (see, e.g., Woodford, 2003), obtained by the firms price setting problem. The real wage gap $\tilde{\omega}_t = \omega_t - \omega_t^{Eff}$, that is, the
gap between the current and the efficient equilibrium real wage, appears on the RHS of the NKPC because it coincides with the marginal cost in our case. Given the linear production function, the difference between the real wage, \( \omega_t \), and the marginal product of labor, \( a_t \), can be expressed as:

\[
mc_t = \omega_t - y_t + l_t = \omega_t - a_t = \bar{\omega}_t. \tag{29}
\]

\( \kappa_p = \frac{(1-\beta_p)(1-\xi_p)}{\xi_p} \) in M1 is the usual expression for the slope of the NKPC.

M2 describes the dynamics of wage inflation, as in Erceg et al. (2000). Symmetrically with respect to the NKPC, the term on the RHS in the wage inflation equation is given by the difference between the marginal rate of substitution between labor and consumption and the real wage, which represents the labor market wedge. Given the utility, (1), and production function, (13), then we:

\[
\begin{align*}
\left[ \frac{v_LL}{v_L} \right] l_t + \left[ \frac{-uCC'}{uC} \right] c_t - \psi_t - \omega_t &= [\sigma x_t - \phi a_t - \psi_t] - \omega_t = \bar{\omega}_t - (\sigma + \phi) x_t, \\
\end{align*}
\]

where \( x_t = y_t - y_t^{Eff} \) denotes the output gap, that is, the gap between actual output and the efficient output, while \( \phi \) and \( \sigma \) are the (assumed constant) elasticity of intertemporal substitution in labor supply and in consumption respectively. \( \kappa_w = \frac{(1-\beta_w)(1-\xi_w)}{\xi_w} \) in M2 is the slope of the wage inflation curve.

Equation M3 is simply the definition of the real wage gap in terms of wage and price inflation and \( \Delta \omega_t^{Eff} = \frac{\omega_t^{Eff} - \omega_{t-1}^{Eff}}{1} \).

The first three equations, M1 – M3, are the same as the ones of a standard New Keynesian model with sticky prices and sticky wages with only ricardian consumers, as in Erceg et al. (2000).\(^1\) Surprisingly, the assumption of having both ricardian and liquidity constrained consumers does not affect the price and wage inflation dynamics. Regarding wage inflation dynamics the key assumption for this result is that the union is maximizing a weighted average of the utility of the two types of agents (see Colciago, 2008).

The IS equation, M4, on the contrary, does depend on \( \lambda \), that is, on the share of liquidity constrained agents in the economy. Appendix A.2.1 shows that M4 derives from the aggregation of the optimal decision of consumption by ricardian agents (i.e.,

\(^{10}\)The only minor difference with Erceg et al. (2000) is in the expression for \( \kappa_w \). This is due to the different assumption regarding the labor market explained in footnote 6.
the Euler equation) and the consumption behavior of liquidity constrained ones, which is simply given by their labor income. With respect to the IS equation of a standard NK model, equation $M4$ features an extra term: $-\frac{\lambda}{1-\phi} E_t \Delta \tilde{\omega}_{t+1}$, i.e. the expected growth of the real wage gap. This is due to the interaction between wage and price staggering and liquidity constrained consumers. The intuition is straightforward. Recall that the real wage determines the consumption of liquidity constrained consumers. The wage gap, then, affects aggregate demand relative to the efficient allocation, through the consumption of liquidity constrained consumers, and $-\frac{\lambda}{1-\phi} E_t \Delta \tilde{\omega}_{t+1}$, hence, enters the IS equation. Indeed, the extra term in the IS disappears if $\lambda = 0$, that is, if there are only ricardian consumers, as in Erceg et al. (2000). Moreover, the extra term can be substituted by the output gap if the nominal wages are flexible, as in Bilbiie (2008). Indeed, in this case, the labor market wedge is nil and therefore from (30) we find,

$$\tilde{\omega}_t - (\sigma + \phi) x_t = 0$$

the wage gap can be rewritten in terms of the output gap. Equation (31) simply states that, by closing the wage gap we can also close the output gap. Then, the IS can be entirely rewritten in terms of output gap as in Bilbiie (2008). Summing up, the extra term, $-\frac{\lambda}{1-\phi} E_t \Delta \tilde{\omega}_{t+1}$, to remain in the IS equation, both ingredients are necessary: (i) price and wage staggering, so that we cannot close the wage gap and the output gap at the same time and therefore the extra term remains in the IS; (ii) liquidity constrained consumers so that, real wage gap affects aggregate demand. In our model these two ingredients are both present, while only (i) is present in Erceg et al. (2000), while only (ii) in Bilbiie (2008). In these two papers, hence, the IS equation is the same as in the standard NK model, without the term $-\frac{\lambda}{1-\phi} E_t \Delta \tilde{\omega}_{t+1}$.

The two blocks of the model are then very evident: the aggregate supply side of the model, $M1 - M3$, derives from the sticky price and wage assumption and do not depend on the assumption of liquidity constrained consumer; the aggregate demand side of the model, $M4$, instead differs from a representative agent economy and characterizes a LAMP economy with sticky wages and prices.

The model just needs to be closed by specifying the behavior of the nominal interest rate, either through monetary policy rules or by optimal policy.
3 Aggregate Dynamics and Determinacy

The most notable result in Bilbiie (2008) is the possibility of an inversion of the slope of the IS curve, that turns positive for sufficiently high values of liquidity constrained consumers. Bilbiie (2008) calls this case: "inverted aggregate demand logic" (IADL). Moreover, he claims that this is an empirically relevant case for standard calibration values. Finally, in the "IADL" region of the parameters space an "inverted Taylor principle" holds: monetary policy needs to be passive for the rational expectation equilibrium to be determinate (i.e., the coefficient on inflation in a Taylor rule should be less than one). This allows Bilbiie (2008) to propose a very interesting reinterpretation of the great inflation vs. great moderation debate.

Here we show that these results are not surviving to the introduction of sticky wages. Appendix A.2.2 shows that in the case of flexible wages the IS equation can be expressed as

$$x_t = E_t x_{t+1} - \frac{(\delta^{fw})^{-1}}{\sigma} E_t \left( r_t - \pi_{t+1} - r^E_{t+1} \right)$$  (32)

where $\delta^{fw} = 1 - \frac{\lambda (\sigma + \phi)}{1 - \lambda} \frac{1}{1 + \phi}$.

As shown by Bilbiie (2008), the slope of the IS becomes positive if the share of non asset holders is high enough. In our case, $\delta^{fw} < 0$ iff $\lambda > (\lambda^*)^{fw} = \frac{1}{1 + \sigma + \phi}$.

The counterpart of (32) in the case of sticky wages is given by

$$x_t = E_t x_{t+1} - \frac{(\delta^{sw})^{-1}}{\sigma} E_t \left( i_t - \pi_{t+1} - r^E_{t+1} \right) + \frac{\lambda}{1 - \lambda} \frac{(\delta^{sw})^{-1}}{1 + \beta + \kappa^w} E_t \left[ \Delta \pi^w_{t+1} - \Delta \hat{\omega}_{t+1} - \beta \left( \Delta \hat{\omega}_{t+1} + \Delta \pi^w_{t+2} \right) \right]$$  (33)

where $\delta^{sw} = 1 - \frac{\lambda (\sigma + \phi)}{1 - \lambda} \frac{\kappa^w}{1 + \beta + \kappa^w}$ represents the counterpart of the slope of the IS with respect to the flexible wage case. Indeed, under flexible wages, i.e., $\xi^w = 0$, then $\kappa^w - \infty$, and $\delta^{sw} - \delta^{fw} = 1 - \frac{\lambda (\sigma + \phi)}{1 - \lambda} \frac{\kappa^w}{1 + \beta + \kappa^w}$. Moreover, the second line in (33) vanishes and we get back to (32).

From the comparison between (32) and (33), we can derive the following two propositions:

**Proposition 1.** The slope of the IS curve under sticky wages, i.e., $\delta^{sw}$, is always greater than the one under flexible wages. Moreover, $\delta^{sw}$ increases with the degree of wage

---

11 The expression is slightly different from Bilbiie (2008) again for our assumption on the labor market.
stickiness.

\[ \delta^{sw} - \delta^{fw} = \frac{\lambda(\sigma+\phi)}{1-\lambda} \left(1 - \frac{\kappa_w}{1+\beta+\kappa_w}\right) = \frac{\lambda(\sigma+\phi)\beta}{1-\lambda(1+\beta+\kappa_w)} > 0. \]

Moreover, \( \frac{\partial \delta^{sw}}{\partial \xi_w} \neq 0 \), it follows that \( \frac{\partial \delta^{sw}}{\partial \xi_w} > 0 \).

Proposition 2. As in the case of flexible wages, also in the case of sticky wages, there exist a threshold value of the share of liquidity constrained consumers beyond which the slopes of the IS schedule, (33), turns positive: \( \lambda > (\lambda^*)^{sw} = \frac{1}{1+(\sigma+\phi)\kappa_w} \). Moreover, \((\lambda^*)^{sw}\) increases with the degree of wage stickiness.

Proof. The proof is trivial since \( \delta^{sw} < 0 \Leftrightarrow \lambda > (\lambda^*)^{sw} = \frac{1}{1+(\sigma+\phi)\kappa_w} \), and \( \frac{\partial (\lambda^*)^{sw}}{\partial \xi_w} = -\left[1 + (\sigma + \phi) \frac{\kappa_w}{1+\beta+\kappa_w}\right]^{-2} \frac{1+\beta}{(1+\beta+\kappa_w)^2} \frac{\partial \kappa_w}{\partial \xi_w} > 0. \)

Also in the case of sticky wages, therefore, there exists a value of \( \lambda \in [0,1] \) such that we can define a "IADL" region. The slope of the IS schedule never changes slope, regardless of the value of \( \lambda \), only in the limit case of fix wages (i.e., \( \xi_w = 1 \implies \kappa_w = 0 \)). However, given that \( \delta^{sw} > \delta^{fw} \), the inversion of the slope of the IS occurs for an higher value of \( \lambda \). In other words, the "IADL" region of the parameter space is smaller when wages are sticky. How much smaller would obviously depend on the calibration.

Corollary. Given Proposition 1 and 2, it follows that sticky wages makes the "IADL" region of the parameter space smaller. Numerically, for realistic calibrations, the "IADL" case is quite implausible when wages are sticky.

Our baseline calibration\(^1\) implies that \((\lambda^*)^{sw} = 0.71\), that is, the model enters the "IADL" region if the share of non asset holders is higher than 71\%, an highly implausible number.\(^2\) In the flexible wage case, instead, \((\lambda^*)^{fw}\) would be equal to 0.16, 4 and half times smaller.

\(^1\)See Section 4.3.1 below. Throughout the current Section, as in the coming Figures, we will use these parameter values.

\(^2\)Moreover, note that we are choosing number against our argument, since we assume high values for \( \sigma = 2 \) and \( \phi = 3 \), and an average duration of wage contracts of 3 quarters. If we were to choose a rather standard alternative calibration, as log-utility in consumption and labor and an average duration for wage contracts of 4 quarters, then \((\lambda^*)^{sw}\) would have been equal to 0.92.
Very interestingly, Bilbiie (2008) shows that the inversion of the slope of the IS curve also determines the inversion of the Taylor principle. More precisely, Bilbiie (2008) demonstrates that when monetary policy follows a simple forward looking Taylor rule as $i_t = \phi_\pi \pi_{t+1}$, then $\phi_\pi$ has to be less than 1 to induce a unique rational expectations equilibria.\textsuperscript{14} Figure 1(i) visualizes this result. It displays the indeterminacy region in the case of flexible prices, as a function of the Taylor rule coefficient, $\phi_\pi$, and of the share of liquidity constrained consumers, $\lambda$. It is evident that as soon the share of non asset holders is above the low threshold value, $(\lambda^*)^{fw}$, the inverted Taylor principle holds. In the case of flexible wages, therefore, "the inverted Taylor principle holds ‘generically’ (i.e., if we exclude some extreme values for some of the parameters)" (Bilbiie, 2008, p. 180).

Given the results in the above Propositions, it is not surprising to verify that under sticky wages the inverted Taylor principle is, instead, confined to extreme values of the parameters space, as shown in Figure 1(ii). For the particular rule studied in Bilbiie (2008) in the case of sticky wages, the exact Taylor principle holds unless the share of non–ricardian consumers assumes very high and implausible values. In other words, sticky wages makes the LAMP assumption irrelevant regarding the Taylor principle,\textsuperscript{14}Bilb\iie (2008) also looks at the robustness of this results, showing that both a contemporaneous Taylor rule and a Taylor rule that responds also to current output somewhat restrict the region of the parameters space where the "inverted Taylor principle" holds. The simple forward looking rule is, however, convenient because it produces a direct link between the inversion of the IS and the inversion of the Taylor principle.
"generically" restoring standard results (see also Colciago, 2008).

(i) Flexible Wages

(ii) Sticky Wages

Figure 2. Indeterminacy regions when $i_t = \phi_\pi \pi_t + \phi_y y_t$.

The above results are generally true, beyond the particular case of the simple forward
looking rule, as Figure 2 shows in the case of a more standard Taylor rule as $i_t = \phi_n \pi_t + \phi_y y_t$. Figure 2(i) displays the flexible wage case determinacy regions for four different values of $\lambda$. As pointed out by Bilbiie (2008), the share of liquidity constrained consumers dramatically changes these regions. In the absence of liquidity constrained consumers, we get the standard Taylor principle in NK models. As soon as $\lambda$ gets above the threshold value, however, the inverted Taylor principle occurs. This is very evident for example comparing 2(i)a), $\lambda = 0$, with 2(i)c), $\lambda = 0.5$. In the case of sticky wages, instead, Figure 2(ii) shows that there is only a minor difference between a LAMP and a standard representative agent NK model regarding the determinacy region. The value of $\lambda$ does not affect the determinacy region in the positive orthant. Moreover, the frontier that corresponds to the standard Taylor principle in NK model (i.e., the almost vertical line) is not affected by the share of non asset holders. Monetary policy needs to be active if wages are sticky.

To conclude, the determinacy region implied by a standard representative agent NK model with sticky prices is basically the same as the one implied by a LAMP economy with sticky prices and sticky wages, unless the share of liquidity constrained consumers assumes high and unrealistic values. When sticky wages are assumed, liquidity constrained consumers substantially do not matter to define the determinacy region for monetary policy rules.

This conclusion shed some shadows on the alternative suggestive interpretation of the great inflation period in Bilbiie (2008). Bilbiie (2008) argues that the economy was in the "IADL" region, because of the low asset market participation in the 70's. Then it was optimal for the Fed to act passively, thereby increasing inflation volatility. While it might be true that the asset market participation was historically low in this period, the labour market rigidity and the wage stickiness were probably historically high.

4 Optimal Monetary Policy

In this Section we will look at the optimal policy problem, cast in the standard linear quadratic framework (see Woodford, 2003).

\footnote{Unfortunately, we were not able to establish analytical results because the dynamics of the model is of third-order. We performed, however, extensive robustness checks available on request.}
4.1 The Welfare Loss Function

We assume that the central bank maximizes a convex combination of the utilities of two types of households, as in Bilbiie (2008). Weights correspond to the relative importance of agents’ groups in the economy. The period economy welfare function reads as:

\[
W_t = \lambda [u(C_{H,t}) - v(L_{H,t})] + (1 - \lambda) [u(C_{S,t}) - v(L_{S,t})]
\]

or

\[
W_t = \lambda u(C_{H,t}) + (1 - \lambda) u(C_{S,t}) - v(L_t)
\]

given that \( L_{H,t} = L_{S,t} = L_t \).

**Proposition 3.** The aggregate welfare loss function approximated at second-order around the efficient steady state described in Section 2.6 is given by:

\[
L = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left( \frac{(\sigma - 1)\lambda}{1 - \lambda} \hat{\omega}^2_t + (\sigma + \phi) \xi^2_t + \frac{\theta_w}{\kappa_w} (\pi^w_t)^2 + \frac{\theta_p}{\kappa_p} \pi^2_t \right). \tag{36}
\]

**Proof.** See Appendix A.3.

The welfare loss function is characterize by four terms. The last three terms are exactly the ones in Erceg et al (2000), and derives from the usual distortions in prices and wages. Indeed, this part of the loss function does not depend on \( \lambda \).

When wages are sticky and a fraction of households \( \lambda \) is liquidity constrained, the welfare loss is characterized by the additional term \( \frac{(\sigma-1)\lambda}{1-\lambda} \hat{\omega}^2_t \). The central bank thus wants to minimize also the real wage gap alongside output gap, wage inflation and price inflation. The wage gap enters the loss function for the same reasons it enters the IS equation (33). When \( \lambda = 0 \), the term \( \hat{\omega}^2_t \) in equation (36) vanishes and the welfare function collapses to the one in Erceg et al (2000).

When wages are flexible, wage inflation obviously does not enter the loss function. Moreover, in contrast to what happens in a sticky wage economy, for the central bank is equivalent to target the output gap or the real wage gap. In this case, the real wage is in fact equal to the marginal rate of substitution. Thus, the relationship between the wage gap and the output gap is:

\[
\hat{\omega}^2_t = (\sigma + \phi)^2 \xi^2_t, \tag{37}
\]
and by closing the output gap the central bank is able to close wage gap at the same time. Therefore, substituting (37) into (36) we obtain (see Appendix A.3.1)

\[ L = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left( (\sigma + \phi) \left( 1 + \frac{(\sigma - 1)(\phi + \sigma)\lambda}{1 - \lambda} \right) x_t^2 + \frac{\theta_p}{\kappa_p} \pi_t^2 \right), \tag{38} \]

which is similar to the one in Bilbiie (2008). (38) collapses to the standard text-book welfare-loss for \( \lambda = 0 \).

4.2 Optimal Responses to Shocks

From the analysis of the model equations (M1) – (M4) and of the loss function (36), and following Erceg et al. (2000), it is immediate to state the following proposition regarding the trade-offs monetary policy is facing.

**Proposition 4.** (i) With sticky wages and prices, it is impossible for monetary policy to attain the optimal social welfare after a technology shock, while it is possible in response to a preference shock.

(ii) If either the prices or the wages are flexible, it would be possible for monetary policy to attain the optimal social welfare also in response to a technology shock.

**Proof.** The proof is straightforward. (i) To attain the optimal social welfare, monetary policy should completely stabilize wage and price inflation, the output gap and the wage gap. Given (M1) and (M2) to stabilize wage and price inflation, monetary policy should contemporaneously close the wage and the output gap. This contradicts equation (M3). In fact, after a technology shock, that affects \( \Delta \omega_t^{Eff} \), the wage gap, price and wage inflation should move to satisfy (M3). Intuition is also straightforward: in the social optimum the real wage follows one-to-one the marginal productivity of labor \( a_t \), but this is simply not possible if the variance of both price and wage inflation is stabilized. Note, instead that the preference shock enters, as usual, only in \( r_t^{Eff} \) so there exists a path for the nominal interest rate that can completely offset the preference shock.

(ii) In the case of flexible wages, the real wage equals the marginal rate of substitution between consumption and labor and the real wage gap can be rewritten in terms of the output gap, i.e., \( \tilde{\omega}_t = (\sigma + \phi) x_t \). This is the reason why wage inflation does not enter the loss function, indeed for the central bank is equivalent
to target the output gap or the real wage gap. But then the model simply becomes isomorphic to the standard NK model:

\[(M1) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa_p (\sigma + \phi) x_t,\]

\[(M4) \quad x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left( i_t - \pi_{t+1} - r_t^{Eff} \right) - \frac{\lambda (\sigma + \phi)}{1 - \lambda} E_t \Delta x_{t+1},\]

with no trade-off for monetary policy in response to a technology shock.

Similarly, in the case of flexible prices, price inflation does not enter the loss function, and the real wage equals the marginal productivity of labor. So the wage gap is zero, and again the model is isomorphic to the standard one:

\[(M2) \quad \pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w (\sigma + \phi) x_t,\]

\[(M4) \quad x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left( i_t - \pi_{t+1} - r_t^{Eff} \right)\]

with no trade-off for monetary policy in response to a technology shock.

The Proposition and the Proof are just restating the well-known result in Erceg et al. (2008), since the endogenous arising of an inflation-output trade-off comes from the supply side of the model which is the same as in Erceg et al. (2008). Hence, we know that the introduction of wage staggering induces an endogenous trade-off for monetary policy. From this perspective, the assumption of a LAMP economy does not changes the monetary policy response from a qualitatively point of view. It may however changes it quantitatively, because of the presence of the future expected change of the wage gap in the IS equation. To have a feeling about how much this difference matters quantitatively, we need to perform some simulations of the optimal policy under commitment.

4.3 Optimal Policy under Commitment

In the presence of a credible commitment, the central bank maximizes the welfare function (36) subject to the following constraints:

\[
\begin{align*}
\pi_t &= \beta E_t \pi_{t+1} + \kappa_p \tilde{\omega}_t \\
\pi_t^w &= \beta E_t \pi_{t+1}^w - \kappa_w \tilde{\omega}_t + \kappa_w (\sigma + \phi) x_t \\
\tilde{\omega}_t &= \tilde{\omega}_{t-1} + \pi_{t+1}^w - \pi_t + \Delta \tilde{\omega}_t^{Eff} \\
\tilde{\omega}_{t-1} &\text{ given.}
\end{align*}
\]  

(39)

The model is solved numerically. The result shows that the approximate Ramsey equilibrium has a recursive nature. The central bank problem determines the path of aggregate
variables. Then the resource constraint of liquidity constrained agents and the definition of aggregate consumption delivers how production is shared between agents. Finally, the IS curve solves for the optimal path of the nominal interest rate.

4.3.1 Calibration

In what follows we will show the optimal impulse response functions (IRFs) of the main economic variables following the technology shock and the preference shock. The model is calibrated as follows.

Preferences. Time is measured in quarters. The discount factor $\beta$ is set to 0.99, so that the annual interest rate is equal to 4 percent. The parameter on consumption in the utility function $\sigma$ is set equal to 2 and the parameters on labour disutility, $\phi$, is set equal to 3.

Production. Following Basu and Fernald (1997), the value added mark-up of prices over marginal cost is set to 0.2. This means that the intermediate goods price elasticity, $\theta_p$ is set equal to 6. The Calvo (1983) probability that firms do not reset prices, $\xi_p$, is set equal to 2/3.

Labour markets. The elasticity of substitution between labor inputs, $\theta_w$ is set equal to 6. The Calvo probability that unions do not reset wages, $\xi_w$, is set equal to 2/3.

Exogenous shocks. We draw the autoregressive coefficient and the standard deviation of the technology shock from Schmitt-Grohé and Uribe (2007), while for what concerns the preference shock we refer to the estimates by Galí and Rabanal (2004). Selected values are $\rho_a = 0.855$, $\sigma_a = 0.0064$, $\rho_\psi = 0.93$ and $\sigma_\psi = 0.025$.

4.3.2 Impulse Response Functions

Here we just focus on the IRFs to a technology shock, since we know that a preference shock features no trade-off for monetary policy, so that monetary policy can completely stabilize the economy as in the standard representative agent model, despite the assumption of LAMP.

Figure 3 shows the effects of a persistent technology shock under the optimal policy. We display deviations from the efficient steady state of the main macroeconomic variables. The monetary policy is endowed with a single instrument, and in this environment it must trade-offs between the two competing distortions due to sticky prices and sticky wages. As a result, the central bank responds to the shock by reducing the
nominal interest rate below steady state for a few quarters, thereby reducing inflation and creating prolonged adjustment of the output gap. Remarkably, in response to an increase in productivity, hours worked fall. The contraction in hours following a positive productivity shock is in line with recent econometric studies using data from the U. S. economy (see, for example, Galí and Rabanal, 2004).

The adjustment, thus, qualitatively resembles the one in a standard NK model with sticky wages and prices, as we argued in the previous Section. Here we want to answer the question at the end of that Section: how the assumption of LAMP changes quantitatively the response of the optimal monetary policy. The IRFs show that varying the degree of asset market participation, $\lambda$, amplifies the transmission of the technology shock on the economy. In particular, the response of ricardian agents' consumption to the technology shock is stronger the larger the share of liquidity constrained agents. The rise in technology leads to lower marginal costs and higher output and brings about an increase in total profits, which is independent of the share of liquidity constrained agents, as other aggregate variables. For this reason lower asset market participation translates into higher dividend income for each ricardian individual, as visualized in the bottom right panel. The higher the number of liquidity constrained agents, the higher the share of individual profits that each asset holder is receiving, and hence the stronger is the reaction of her consumption. To support such an outcome the Euler equation requires lower asset market participation to be associated with more aggressive cuts of the nominal interest. Then, the stronger would be the increase in output. The larger increase in labor demand causes a larger increase in the real wage and a lower decrease in hours worked, hence, a greater income and consumption of liquidity constrained agents.

The main point, however, is that these effects are very minor. The paths of the variables of interest for the optimal monetary policy problem, i.e., the different gaps and distortions, are not very sensitive to changes in the share of liquidity constrained consumers. All in all, also from a quantitative point of view, the optimal policy response of a NK models with price and wage stickiness is, therefore, only marginally affected by the assumption of a LAMP economy.16

---

16The response of the efficient level of output is somewhat in between the responses in the top left panel of Figure 3. Hence the output gap switches sign from negative to positive as $\lambda$ changes, but this effect is quantitatively negligible.
Moreover, when $\sigma = 1$, the LAMP hypothesis has no effect at all on the optimal monetary policy response, as shown in Figure 4. In this case, neither the objective function nor the constraints depend on the share of liquidity constrained agents. Thus, in response to shocks, the optimal policy implements the same equilibrium path for the welfare relevant variables as in a full participation economy. In this case, society welfare will not be affected by the presence of liquidity constrained agents and just the interest rate will be affected by LAMP assumption through the IS curve. Obviously, the two types of agents will be affected in a different way from the optimal response of monetary policy.
To further stress our main point, next we briefly consider the effect of a cost-push shock under the baseline calibration. This is not interesting per se, since our model does not need to assume this non microfounded shock to deliver an endogenous trade-off, but rather for the sake of comparison with Bilbiie (2008). Figure 5 shows that in response to a cost push shock the dynamic adjustments of the variables under optimal policy is quite sensitive to variations in the value of $\lambda$, in line with Bilbiie (2008). However, as we showed above, this is not the case for the microfounded shocks, as the technology shock.

Figure 4. IRFs to a technology shock under optimal policy, in the case $\sigma = 1$. 
Finally Table 1 shows the unconditional welfare loss under optimal policy for alternative parameterizations of the two prominent features of our model economy: the share of liquidity constrained agents and the average durations of wage contracts (i.e., $(1 - \xi_w)^{-1}$). The unconditional welfare loss is expressed in terms of percentage of the efficient steady state level of consumption, and it is calculated plugging the unconditional variances into (36). As we know from Proposition 4, in the case of flexible wages (i.e., $\xi_w = 0 \Rightarrow (1 - \xi_w)^{-1} = 1$), monetary policy is able to attain the social optimum. The unconditional welfare loss is not surprisingly increasing as the two distortions get larger.

5 Optimal Simple Rules

In this section we evaluate whether LAMP affects the design of optimal rules in the model outlined above. Following Schmitt-Grohé and Uribe (2007), we require the rules to be simple and implementable. The implementability condition requires policies to deliver local uniqueness of the rational expectations equilibrium. Simplicity means restricting
attention to rules whereby the nominal interest rate is a function of a small number of easily observable macroeconomic variables. Moreover, we limit attention to policy coefficients in the interval [-10, 10]. The reason for which we select this interval is that larger coefficients response would fall out of any plausible estimate and would have little credibility.\footnote{Furthermore, as we will see, in our small scale model, very often the optimal coefficients will take the limit values. Extending the interval however, will produce very large coefficients, but with only extremely negligible, if not zero, gains in welfare. This is because the loss function is basically flat after some large values of the coefficients.} Finally, we want to study how the two main features of the model (LAMP and wage stickiness) affects the optimal simple rules, so we will look at different values of $\lambda$ and $\xi_w$.

We will just show the two main results from our analysis, referring to Table 2 and 3 respectively.\footnote{A more extended analysis is available upon request.}

**Result 1.** In the case of pure inflation targeting rules (see Table 2), the optimal rule calls for a strong response of monetary policy. The LAMP assumption makes the optimal rule highly passive if wages are flexible. However, if wages are sticky, the optimal rule is restored to be highly active, as in the standard NK model.

As in Bilbiie (2008), Table 2 focuses on two simple inflation targeting rule: contemporaneous and forward-looking. Result 1 simply establishes that the same logic

<table>
<thead>
<tr>
<th>Average duration of wage contracts</th>
<th>$\lambda = 0$</th>
<th>$\lambda = 0.25$</th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{1-\xi_w}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Full Commitment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.0046</td>
<td>0.0054</td>
<td>0.007</td>
<td>0.0108</td>
</tr>
<tr>
<td>3</td>
<td>0.0059</td>
<td>0.0066</td>
<td>0.008</td>
<td>0.0125</td>
</tr>
<tr>
<td>4</td>
<td>0.0066</td>
<td>0.0071</td>
<td>0.0084</td>
<td>0.0125</td>
</tr>
<tr>
<td>5</td>
<td>0.0069</td>
<td>0.0075</td>
<td>0.0086</td>
<td>0.0124</td>
</tr>
</tbody>
</table>

Table 1: Unconditional welfare loss under full commitment.

We consider alternative parameterizations for the share of non ricardian consumers and alternative average duration of wage contracts. The welfare cost is expressed as a percentage of the efficient steady state level of consumption, while the mean duration of wage contracts is expressed in quarters.
Table 2: PANEL A: Optimal contemporaneous inflation response coefficient (left), Welfare Loss (RIGHT). PANEL B: Optimal expected inflation response coefficient (left), Welfare Loss (RIGHT). We consider alternative parameterizations for the share of non-Ricardian consumers and alternative average duration of wage contracts. The welfare cost is expressed as a percentage of the efficient steady state level of consumption, while the mean duration of wage contracts is expressed in quarters.
of the previous Sections also apply to these optimal rules: sticky wages offset the effects of LAMP on the optimal monetary policy response. Consider the pure current inflation targeting rule in Table 2. In a fully Ricardian economy ($\lambda = 0$) with flexible wages ($((1 - \xi_w)^{-1} = 1$) the optimal response coefficient implies a strong anti-inflationary stance, because in the absence of inflation-output gap trade-off, stabilizing inflation also results in output stabilization. In our exercise, thus, the inflation coefficient hits the upper bound (i.e. $\phi_\pi = 10$). Removing the upper bound on policy parameters would result in an unbounded inflation coefficient response and zero welfare loss. The optimal rule is extremely effective, as it delivers a welfare loss equal to 0.002 percent of steady state consumption. These results resembles those in other studies such as Schmitt-Grohé and Uribe (2007).

Introducing LAMP in this environment has dramatic consequences for the design of optimal interest rate rules. The optimal contemporaneous rule turns passive and features a strongly negative inflation response, indeed $\phi_\pi$ hits the lower bound equal to -10. We are in a "IADL" region and the central bank should move the nominal rate such that the real rate decreases when asset market participation is limited enough. It is worth emphasizing that the negative inflation coefficient obtained under LAMP and flexible wages does not merely serve the purpose of ensuring the uniqueness of the rational expectations equilibrium. Under the contemporaneous rule a very strong response to inflation would, in fact, guarantee determinacy in the LAMP economy. However, it would deliver a lower welfare with respect to the passive rule considered here.

Let us consider now what happens when sticky wages come into the picture: even a very low, and below estimates, degree of wage stickiness restores the optimality of an active rule for any empirically plausible share of liquidity constrained agents. When the degree of wage stickiness assumes values compatible with the empirical evidence the optimal policy is highly active no matter the extent to which we limit asset market participation. Again, wage stickiness limits the likelihood of a reversal in the slope of the IS curve and it restores standard policy prescriptions. In other words, once wage stickiness is considered, LAMP has just minor quantitative implications for the design of optimal monetary policy, as the optimal policy calls for a stronger reaction to inflation as the share of liquidity constrained agents increases.

Similar considerations extend to the forward looking pure inflation targeting rule in Table 2. As in a standard economy, the simple rules considered here perform quite well
in terms of welfare even in the presence of liquidity constrained agents. The welfare loss gets relevant just in the case wage stickiness is coupled with an implausibly large share of liquidity constrained consumers. However, this is partly due to the fact that we restrict the interval of admissible values for $\phi_\pi$.

We now turn to the second result, from the analysis of Table 3.

**Result 2.** Table 3 establishes the following: (i) Result 1 is robust to the case of hybrid rules; (ii) a rule targeting both price and wage inflation delivers a welfare loss very close to the one under optimal policy; (iii) responding to output only marginally improves the performance of a pure targeting rule.

In Table 3 we consider the performance of other two popular rules in the literature. The first features both a response to inflation and to deviations of output from the steady state, while the second replaces output with wage inflation. Results 1 is confirmed. The inverted Taylor principle is again a feature of the economy with LAMP and wage flexibility. Nominal wage stickiness calls instead for an aggressive response to both price and wage inflation.

In line with Erceg et al. (2000), responding to wage inflation substantially reduces the welfare loss with respect to the rules considered in Table 2, and gets remarkably close to the optimal policy welfare level. Moreover, the relative magnitude of the two coefficients depend on the relative degree of stickiness between the two nominal variables. The larger between the two coefficients is the one multiplying the inflation of the stickier variable between prices and wages. Moreover, both coefficients are increasing in the share of LAMP, and are generally very large (possibly unbounded in the case of wage inflation targeting for high degree of wage stickiness). It follows that for realistic values of the degree of wage stickiness, this rule calls for complete wage stabilization.

Finally, in line with Schmitt-Grohé and Uribe (2007), responding to output is not a desirable feature for an optimal rule.
### Average duration of wage contracts

\[
(1-\xi_w)^{-1}
\]

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i_t = \phi^\pi \pi_t + \phi^y y_t)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.04, 3.4*10^{-4}</td>
<td>-10, 0.035, 2.4*10^{-4}</td>
<td>-10, 0.035, 2.4*10^{-4}</td>
<td>-10, 0.034, 2.28*10^{-4}</td>
</tr>
<tr>
<td>2</td>
<td>5.08, 0.14, 0.012</td>
<td>5.21, 0.11, 0.011</td>
<td>6.19, 0.06, 0.009</td>
<td>-10, -0.11, 0.036</td>
</tr>
<tr>
<td>3</td>
<td>4.42, 0.15, 0.019</td>
<td>4.7, 0.12, 0.018</td>
<td>5.59, 0.07, 0.015</td>
<td>10, -0.07, 0.014</td>
</tr>
<tr>
<td>4</td>
<td>4.47, 0.15, 0.026</td>
<td>4.79, 0.13, 0.024</td>
<td>5.64, 0.08, 0.021</td>
<td>9.43, -0.04, 0.019</td>
</tr>
<tr>
<td>5</td>
<td>4.68, 0.15, 0.032</td>
<td>5.03, 0.13, 0.03</td>
<td>5.87, 0.09, 0.027</td>
<td>9.29, -0.03, 0.024</td>
</tr>
</tbody>
</table>

| \(i_t = \phi^\pi \pi_t + \phi^\pi_l \pi_l\) |       |       |       |       |
| 1                 | 0, 0.012, 3.5*10^{-4} | -10, 0.12, 2.3*10^{-4} | -10, 0.17, 2.4*10^{-4} | -10, 0.03, 2.4*10^{-4} |
| 2                 | 10, 7.15, 0.005 | 10, 5.27, 0.005 | 10, 4.10, 0.006 | 10, 8.66, 0.011 |
| 3                 | 6.67, 10, 0.006 | 7.83, 10, 0.007 | 10, 10, 0.008 | 10, 9.02, 0.012 |
| 4                 | 4.26, 10, 0.007 | 5.19, 10, 0.008 | 7, 10, 0.009 | 10, 10, 0.013 |
| 5                 | 3.21, 10, 0.008 | 4.05, 10, 0.008 | 5.77, 10, 0.009 | 10, 10, 0.013 |

Table 3: PANEL A: Optimal inflation response coefficient (left), Optimal output response coefficient (centerl), Welfare Loss (right). Optimal inflation response coefficient (left), Optimal wage inflation response coefficient (centerl), Welfare Loss (right). We consider alternative parameterizations for the share of non ricardian consumers and alternative average duration of wage contracts. The welfare cost is expressed as a percentage of the efficient steady state level of consumption, while the mean duration of wage contracts is expressed in quarters.
6 Conclusions

In this paper, we study optimal monetary policy and optimal simple rules in a NK model with nominal wage stickiness and LAMP. To the best of our knowledge this is the first attempt to study optimal monetary policy in this particular framework. We find some new results: 1) the welfare-based loss function depends on real wage gap beside on the output gap, wage inflation and price inflation; 2) the slope of the IS curve is always greater than one and the effectiveness of monetary policy increases as nominal wage stickiness increases; 3) the inversion of the slope of the IS occurs only for very high value of LAMP; 4) liquidity constrained consumers substantially do not matter to define the determinacy region for monetary policy rules; 5) surprisingly, LAMP does not matter much for the design of optimal monetary policy once nominal wage rigidity, an uncontroversial empirical fact, is taken into account. The estimates on the fraction of liquidity consumers obtained in some recent empirical works (Di Bartolomeo et al 2010, Coenen and Straub 2005, and Forni et al 2009), strongly support our results.
References


Di Bartolomeo, G., L. Rossi, and M. Tancioni (forthcomings). Monetary policy, rule of thumb consumers and external habits: A g7 comparison. *Applied Economics*.


A Technical Appendix

A.1 Derivation of the Efficient Equilibrium Output

In this section we solve the Social Planner problem (SPP). The equilibrium output which solve the SPP corresponds to efficient equilibrium output. The SPP reads as

\[
\max_{\{C_{H,t},C_{S,t},L_t\}} \lambda \frac{\Psi_t C_{H,t}^{1-\sigma}}{1-\sigma} + (1 - \lambda) \frac{\Psi_t C_{S,t}^{1-\sigma}}{1-\sigma} - \lambda \frac{L_{H,t}^{1+\phi}}{1+\phi} - (1 - \lambda) \frac{L_{S,t}^{1+\phi}}{1+\phi}
\]  

s.t

\[
C_t = Y_t = A_t L_t = \lambda C_{H,t} + (1 - \lambda) C_{S,t} = A_t (\lambda L_{H,t} + (1 - \lambda) L_{S,t})
\]

Forming the Lagrangian \(\mathcal{L}\), the problem can be rewritten as

\[
\max_{\{C_{H,t},C_{S,t},L_t\}} \mathcal{L} = \lambda \frac{\Psi_t C_{H,t}^{1-\sigma}}{1-\sigma} + (1 - \lambda) \frac{\Psi_t C_{S,t}^{1-\sigma}}{1-\sigma} - \lambda \frac{L_{H,t}^{1+\phi}}{1+\phi} - (1 - \lambda) \frac{L_{S,t}^{1+\phi}}{1+\phi} - \mu_t [\lambda C_{H,t} + (1 - \lambda) C_{S,t} - A_t (\lambda L_{H,t} + (1 - \lambda) L_{S,t})]
\]

The FOCs with respect to consumption levels are:

\[
\frac{\partial \mathcal{L}}{\partial C_{H,t}} = 0 : \lambda \Psi_t C_{H,t}^{-\sigma} = \mu_t \lambda
\]

\[
\frac{\partial \mathcal{L}}{\partial C_{S,t}} = 0 : (1 - \lambda) \Psi_t C_{S,t}^{-\sigma} = \mu_t (1 - \lambda)
\]

which imply

\[
C_{H,t}^{-\sigma} = C_{S,t}^{-\sigma} = C_t^{-\sigma}
\]

At the efficient equilibrium agents have the same marginal utility of consumption. Since consumers have identical preferences it follows that they also have the same level of consumption: \(C_{H,t} = C_{S,t} = C_t\). FOCs with respect to the labor supply are as follows

\[
\frac{\partial \mathcal{L}}{\partial L_{H,t}} = 0 : \lambda L_{H,t}^{\phi} = \mu_t A_t \lambda
\]

\[
\frac{\partial \mathcal{L}}{\partial L_{S,t}} = 0 : (1 - \lambda) L_{S,t}^{\phi} = \mu_t A_t (1 - \lambda)
\]

Combining the latter equations delivers

\[
L_{H,t}^{\phi} = L_{S,t}^{\phi} = L_t^{\phi}
\]

which implies that at the efficient equilibrium agents work for the same amount of hours \(L_{H,t} = L_{S,t} = L_t\). In short, at the efficient equilibrium the economy behaves as if there
was a representative agent with marginal rate of substitution between consumption and hours given by $\Psi_t^{-1} C_t^\sigma L_t^\phi$. The social planner sets the latter equal to the marginal product of labor, $A_t$, which also represents the equilibrium real wage, $(W/P)^{Eff}_t$. Using the relationship just described, imposing the market clearing condition $Y_t = C_t$ and using the production function, delivers the efficient level of output as

$$Y^{Eff}_t = A_t^{1+\phi} \Psi_t^{1+\sigma} \quad (46)$$

Log-linearizing and considering that $\Psi = 1$ delivers the log-deviations of the efficient level of output from the efficient steady state as

$$y^{Eff}_t = \frac{1 + \phi}{\sigma + \phi} a_t + \frac{1}{\sigma + \phi} \psi_t. \quad (47)$$

In the efficient equilibrium the Euler equation for Ricardian must be satisfied. Since the consumption is equal for the two class of agents, then the Euler equation must be satisfied by output. The natural rate of interest is therefore:

$$r^{Eff}_t = \sigma \Delta y^{Eff}_t - \Delta \psi_{t+1}. \quad (48)$$

## A.2 Derivation of the log-linear model

[TO BE COMPLETED]

### A.2.1 Derivation of the IS curve

Log-linearization of the Euler equation of Ricardian agents leads

$$c_{s,t} = E_t c_{s,t+1} - \frac{1}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{1}{\sigma} \Delta \psi_{t+1}$$

while from the consumption function of rule-of-tumb consumer we get:

$$c_{H,t} = l_t + \omega_t$$

while aggregate consumption is

$$c_t = (1 - \lambda) c_{s,t} + \lambda c_{H,t}$$

then solving the latter equation for $c_{s,t}$

$$c_{s,t} = \frac{1}{1 - \lambda} c_t - \frac{\lambda}{1 - \lambda} c_{H,t}$$
substituting in the Euler equation we get
\[
\frac{1}{1-\lambda}c_t - \frac{\lambda}{1-\lambda}c_{H,t} = E_t \left( \frac{1}{1-\lambda}c_{t+1} - \frac{\lambda}{1-\lambda}c_{H,t+1} \right) - \frac{1}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{1}{\sigma} \Delta \psi_{t+1}
\]

or
\[
c_t = E_t (c_{t+1} - \lambda \Delta c_{H,t+1}) - \frac{(1-\lambda)}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{(1-\lambda)}{\sigma} \Delta \psi_{t+1}
\]
or substituting for \( c_t = y_t \) and for \( E_t \Delta c_{H,t+1} = E_t (\Delta l_{t+1} + \Delta \omega_{t+1}) \)
\[
y_t = E_t y_{t+1} - \lambda E_t \Delta l_{t+1} - \lambda E_t \Delta \omega_{t+1} - \frac{(1-\lambda)}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{(1-\lambda)}{\sigma} \Delta \psi_{t+1}
\]
given the aggregate production function
\[
y_t = l_t + a_t
\]
then we substitute \( l_t \) for \( l_t = y_t - a_t \), then we get
\[
y_t = E_t y_{t+1} - \lambda E_t \Delta y_{t+1} + \lambda E_t \Delta a_{t+1} - \lambda E_t \Delta \omega_{t+1} - \frac{(1-\lambda)}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{(1-\lambda)}{\sigma} \Delta \psi_{t+1}
\]
or solving for \( y_t \)
\[
y_t = E_t y_{t+1} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1} - \frac{1}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{1}{\sigma} \Delta \psi_{t+1}
\]
rewriting equation (49) in terms of output gap from the efficient equilibrium output we get:
\[
y_t - y_t^{Eff} = E_t \left( y_{t+1} - y_t^{Eff} \right) + \Delta y_t^{Eff} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1} - \frac{1}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{1}{\sigma} \Delta \psi_{t+1}
\]
we define \( x_t = y_t - y_t^{Eff} \), then
\[
x_t = E_t x_{t+1} + \Delta y_t^{Eff} + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1} - \frac{1}{\sigma} E_t (i_t - \pi_{t+1}) - \frac{1}{\sigma} \Delta \psi_{t+1}
\]
We now want to rewrite the IS curve, in terms of the efficient real rate of interest. Recall that \( r_t^{Eff} = \sigma \Delta y_t^{Eff} - \Delta \psi_{t+1} \), hence
\[
x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left( i_t - \pi_{t+1} - r_t^{Eff} \right) + \frac{\lambda}{1-\lambda} E_t \Delta a_{t+1} - \frac{\lambda}{1-\lambda} E_t \Delta \omega_{t+1}.
\]
Given the definition of the real wage gap \( \tilde{\omega}_t = \omega_t - \omega_t^{Eff} \), we can finally write the IS as
\[
x_t = E_t x_{t+1} - \frac{1}{\sigma} E_t \left( i_t - \pi_{t+1} - r_t^{Eff} \right) - \frac{\lambda}{1-\lambda} E_t \Delta \tilde{\omega}_{t+1}.
\]
A.2.2 The slope of the IS curve

Flexible wages Find $\Delta \omega_{t+1}$ and substitute into (49). In the case of flexible wages the real wage is given by

$$\omega_t = \sigma c_t + \phi y_t + \psi_t,$$

$$mc_t = \omega_t - (y_t - l_t) = \omega_t - a_t,$$

$$mc_t = (\sigma + \phi) x_t.$$  \hfill (54)

Hence

$$\Delta \omega_{t+1} = (\sigma + \phi) \Delta x_{t+1} + \Delta a_{t+1}$$

Substitute in (51) to get

$$x_t = E_t x_{t+1} - \frac{(\delta w)^{-1}}{\sigma} E_t \left( r_t - \pi_{t+1} - \pi_{t+1}^{Eff} \right)$$ \hfill (55)

where $\delta w = 1 - \lambda \frac{1}{1-\lambda} (\sigma + \phi)$.

Sticky wages Find $\Delta \omega_{t+1}$ and substitute into (49). In the case of sticky wages the real wage is given by

$$\omega_t = \frac{1}{1 + \beta + \kappa_w} [w_{t-1} - p_t] + \frac{\beta}{1 + \beta + \kappa_w} E_t (w_{t+1} - p_t) + \frac{\kappa_w}{1 + \beta + \kappa_w} ((\sigma + \phi) x_t + a_t)$$ \hfill (56)

This is a weighted average between: (i) the past nominal wage at current prices; (ii) the future nominal wage at current prices; (iii) the flexible wage ($mc + a$). Note that as $\xi_w > 0$, then $\kappa_w > \infty$, and this expression collapses to the usual flexible wage case.

Then

$$\Delta \omega_{t+1} = \frac{1}{1 + \beta + \kappa_w} [\Delta w_t - \Delta p_{t+1}] + \frac{\beta}{1 + \beta + \kappa_w} E_t (\Delta w_{t+2} - \Delta p_{t+1})$$

$$+ \frac{\kappa_w}{1 + \beta + \kappa_w} ((\sigma + \phi) \Delta x_{t+1} + \Delta a_{t+1})$$

$$\Delta \omega_{t+1} = F + \frac{\kappa_w}{1 + \beta + \kappa_w} ((\sigma + \phi) \Delta x_{t+1} + \Delta a_{t+1})$$

Substitute that into (51) to get

$$x_t = E_t x_{t+1} - \frac{(\delta w)^{-1}}{\sigma} E_t \left( r_t - \pi_{t+1} \right) - \frac{(\delta w)^{-1}}{\sigma} \Delta \psi_{t+1} + \frac{(\delta w)^{-1}}{\sigma} \Delta \gamma_{t+1}^{Eff} +$$

$$\frac{\lambda}{1 - \lambda} \frac{(\delta w)^{-1}}{1 + \beta + \kappa_w} \left\{ (1 + \beta) E_t \Delta a_{t+1} - E_t \left[ (\pi^w_t - \pi_{t+1}) + (\pi^w_{t+2} - \pi_{t+1}) \right] \right\}$$ \hfill (57)
where $\delta^w = 1 - \frac{\lambda}{1-\lambda} \frac{\kappa_w (\sigma + \phi)}{1 + \beta + \kappa_w}$. Finally note that

$$(1 + \beta) E_t \Delta \omega_{t+1} - E_t \left[ (\pi_t^w - \pi_{t+1}) + \beta (\pi_{t+2}^w - \pi_{t+1}) \right] = E_t \left[ \Delta \pi_{t+1}^w - \Delta \omega_{t+1} - \beta (\Delta \omega_{t+1} + \Delta \pi_{t+2}^w) \right].$$

**A.3 Derivation of the Welfare-based Loss Function**

In order to derive a second-order approximation of the household’s utility function, we assume that the steady state of our economy is efficient. Under this assumption, we have that in the steady state:

$$\frac{v_{L,H}}{u_{C,H}} = \frac{v_{L,S}}{u_{C,S}} = \frac{W}{P} = \frac{Y}{L} = 1 \quad (59)$$

where $L_H = L_S = L = Y$ and $C_H = C_S = C = Y$. The last equality in (59) holds since the economy production function is: $Y_t = L_t A_t$, where $A = 1$ in steady state. As shown in section once we get the efficient steady firms profits are zero in the steady state and the two households budget constraint is identical, so that $C_S = C_L = C$.

In order to derive a second order approximation of the households utility function, as in Bilbiie (2008) we assume that the Central Bank maximizes a convex combination of the utilities of two types of households, weighted by the mass of agents of each type, i.e.:

$$W_t = \lambda \left[ u(C_{H,t}) - v(L_{H,t}) \right] + (1 - \lambda) \left[ u(C_{S,t}) - v(L_{S,t}) \right] \quad (60)$$

we know that in our model $L_{H,t} = L_{S,t} = L_t$ for each $t$, this means that (60) can be rewritten as

$$W_t = \lambda u(C_{H,t}) + (1 - \lambda) u(C_{S,t}) - v(L_t) \quad (61)$$

A second order approximation of $\lambda u(C_{H,t})$ delivers

$$\lambda u(C_{H,t}) \approx \lambda \left[ u(C_H) + u_{C_H} (C_{H,t} - C_H) + u_\psi (\psi_t - \psi) \right] +$$

$$+ \frac{\lambda}{2} \left[ u_{C_H} C_H (C_{H,t} - C_H)^2 + 2 u_{C_H} \psi (C_{H,t} - C_H) (\psi_t - \psi) + u_{\psi \psi} (\psi_t - \psi)^2 \right]$$

or

$$\lambda u(C_{H,t}) \approx \lambda \left[ u(C_H) + u_{C_H} C_H \left( c_{h,t} + \frac{1}{2} \sigma^2_{h,t} \right) + u_\psi \psi \left( \psi_t + \frac{1}{2} \psi_t^2 \right) \right] +$$

$$+ \frac{\lambda}{2} \left[ u_{C_H} C_H^2 c_{h,t}^2 + 2 u_{C_H} \psi c_{h,t} \psi_t + u_{\psi \psi} \psi^2 \psi_t^2 \right]$$

37
\[ \lambda u(C_{H,t}) - \lambda u(C_H) \simeq \lambda u_{C_H} C_H \left( c_{h,t} + \frac{1}{2} c_{h,t}^2 \right) + \frac{\lambda}{2} u_{C_H} C_H C_H^2 c_{h,t}^2 + \lambda u_{C_H} C_H \psi c_{h,t} \psi_t \]
\[ + \lambda u_{C_H} C_H \psi_c h_t \psi_t^2 + \frac{\lambda}{2} u_{C_H} C_H \psi c_{h,t} \psi_t^2 \left( \psi_t^4 + \frac{1}{2} \psi_t^2 \right) \]

\[ \lambda u(C_{H,t}) - \lambda u(C_H) \simeq \lambda u_{C_H} C_H \left( c_{h,t} + \frac{1}{2} c_{h,t}^2 \right) + \frac{\lambda}{2} u_{C_H} C_H C_H^2 c_{h,t}^2 + \lambda u_{C_H} C_H \psi c_{h,t} \psi_t + \text{tip} \]

\[ \lambda u(C_{H,t}) - \lambda u(C_H) \simeq \lambda u_{C_H} C_H \left( c_{h,t} + \frac{1}{2} c_{h,t}^2 \right) + \frac{\lambda}{2} u_{C_H} C_H C_H^2 c_{h,t}^2 + \lambda u_{C_H} C_H \psi c_{h,t} \psi_t + \text{tip} \]

notice that
\[
\frac{\lambda}{2} u_{C_H} C_H C_H^2 c_{h,t}^2 + \lambda u_{C_H} C_H \psi c_{h,t} \psi_t^2 = \lambda u_{C_H} C_H \left( \frac{1}{2} u_{C_H} C_H C_H^2 c_{h,t}^2 + \frac{u_{C_H} \psi \psi}{u_{C_H} c_{h,t} \psi_t} \right)
\]

since \( \frac{u_{C_H} \psi}{u_{C_H} C_H} = -\sigma \)

\[
\frac{\lambda}{2} u_{C_H} C_H C_H^2 c_{h,t}^2 + \lambda u_{C_H} C_H \psi c_{h,t} \psi_t = \lambda u_{C_H} C_H \left( -\frac{1}{2} \sigma c_{h,t}^2 + \frac{u_{C_H} \psi \psi}{u_{C_H} c_{h,t} \psi_t} \right)
\]

thus
\[
\lambda u(C_{H,t}) - \lambda u(C_H) \simeq \lambda u_{C_H} C_H \left( c_{h,t} + \frac{1}{2} (1 - \sigma) c_{h,t}^2 + \frac{u_{C_H} \psi \psi}{u_{C_H} c_{h,t} \psi_t} \right) + \text{tip}
\]

Given the assumed functional form
\[
\frac{u_{C_H} \psi \psi}{u_{C_H} C_H} = 1
\]

\[
\lambda u(C_{H,t}) - \lambda u(C_H) \simeq \lambda u_{C_H} C_H \left( c_{h,t} + \frac{1}{2} (1 - \sigma) c_{h,t}^2 + c_{h,t} \psi_t \right) + \text{tip}
\]

Similarly a second order approximation to the utility of ricardian agents

\[
(1 - \lambda) u(C_{s,t}) - \lambda u(C_s) \simeq (1 - \lambda) u_{C_s} C_s \left( c_{s,t} + \frac{1}{2} (1 - \sigma) c_{s,t}^2 + c_{s,t} \psi_t \right) + \text{tip}
\]

Also a second order approximation to \( v(L_t) \) yields:

\[
v(L_t) - v(L) \simeq v_{L,L} \left( L_t + \frac{1 + \phi L_t^2}{2} \right)
\]

(62)

Summing all the terms

\[
W_t - W = \lambda u_{C_H} C_H \left( c_{h,t} + \frac{1}{2} (1 - \sigma) c_{h,t}^2 + c_{h,t} \psi_t \right) + \]
\[ + (1 - \lambda) u_{C_s} C_s \left( c_{s,t} + \frac{1}{2} (1 - \sigma) c_{s,t}^2 + c_{s,t} \psi_t \right) - v_{L,L} \left( L_t + \frac{1 + \phi L_t^2}{2} \right)
\]

38
\[
W_t - W = \lambda u_{C_H} C_H \left( c_{h,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{h,t}^2 + c_{h,t} \psi_t \right) + \\
+ (1 - \lambda) u_{C_s} C_s \left( c_{s,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{s,t}^2 + c_{s,t} \psi_t \right) - v_L L \left( l_t + \frac{1 + \phi l_t^2}{2} \right)
\]

Given our assumptions, steady state consumption levels are identical as well as hours worked, in this case

\[
W_t - W = \lambda u_{C_C} \left( c_{h,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{h,t}^2 + c_{h,t} \psi_t \right) + \\
+ (1 - \lambda) u_{C_C} C_t \left( c_{s,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{s,t}^2 + c_{s,t} \psi_t \right) - v_L L \left( l_t + \frac{1 + \phi l_t^2}{2} \right)
\]

or

\[
W_t - W = \lambda u_{C_C} \left( c_{h,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{h,t}^2 \right) + u_{C_C} C_t \psi_t \\
+ (1 - \lambda) u_{C_C} C_t \left( c_{s,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{s,t}^2 \right) - v_L L \left( l_t + \frac{1 + \phi l_t^2}{2} \right) + \text{tip}
\]

From the economy production function we know that

\[ l_t = y_t + d_{w,t} + d_{p,t} - a_t \]

where \( d_{w,t} = \log \int_0^1 \left( \frac{W_{j,t}}{W_t} \right)^{-\theta_w} \, dj \) is the log of the wage dispersion and \( d_{p,t} = \log \int_0^1 \left( \frac{P_i}{P_t} \right)^{-\theta_p} \, di \) is the log of the price dispersion. Both terms are of second order and therefore they cannot be neglected in a second order approximation. Notice that

\[ l_t^2 = (y_t + d_{w,t} + d_{p,t} - a_t)^2 = y_t^2 + a_t^2 - 2y_ta_t \]

thus

\[
W_t - W = \lambda u_{C_C} \left( c_{h,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{h,t}^2 \right) + u_{C_C} C_t \psi_t \\
+ (1 - \lambda) u_{C_C} C_t \left( c_{s,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{s,t}^2 \right) - \\
- v_L L \left( y_t + d_{w,t} + d_{p,t} - a_t + \frac{1 + \phi}{2} (y_t^2 + a_t^2) \right) + \text{tip}
\]

or

\[
W_t - W = \lambda u_{C_C} \left( c_{h,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{h,t}^2 \right) + u_{C_C} C_t \psi_t \\
+ (1 - \lambda) u_{C_C} C_t \left( c_{s,t} + \frac{1}{2} \left( 1 - \sigma \right) c_{s,t}^2 \right) - \\
- v_L L \left( y_t + d_{w,t} + d_{p,t} - a_t + \frac{1 + \phi}{2} y_t^2 \right) - (1 + \phi) y_t a_t + \text{tip}
\]
or
\[
W_t - W = \lambda u_CC \left( c_{h,t} + \frac{1}{2} (1 - \sigma) c_{h,t}^2 \right) + u_CC c_{h,t} \psi_t \\
+ (1 - \lambda) u_CC \left( c_{s,t} + \frac{1}{2} (1 - \sigma) c_{s,t}^2 \right) \\
- v_L L \left( y_t + d_{w,t} + d_{p,t} - a_t + \frac{1 + \phi}{2} y_t^2 - (1 + \phi) y_t a_t \right) + tip
\]
or, since \( u_CC = v_L L \)
\[
\frac{W_t - W}{u_CC} = c_t + \frac{\lambda}{2} (1 - \sigma) c_{h,t}^2 + \frac{(1 - \lambda)}{2} (1 - \sigma) c_{s,t}^2 + u_CC c_t \psi_t + \\
- \left( y_t + d_{w,t} + d_{p,t} - a_t + \frac{1 + \phi}{2} y_t^2 - (1 + \phi) y_t a_t \right) + tip
\]
then using equilibrium condition \( c_t = y_t \)
\[
\frac{W_t - W}{u_CC} = y_t + \frac{(1 - \sigma)}{2} \left[ \lambda c_{h,t}^2 + (1 - \lambda) c_{s,t}^2 \right] + c_t \psi_t + \\
- \left( y_t + d_{w,t} + d_{p,t} - a_t + \frac{1 + \phi}{2} y_t^2 - (1 + \phi) y_t a_t \right) + tip
\]
Next notice that
\[
\hat{c}_{H,t} = w_t + l_t
\]
then
\[
c_{H,t}^2 = w_t^2 + l_t^2 + 2 w_t l_t \\
= w_t^2 + \hat{y}_t^2 + a_t^2 - 2 y_t a_t + 2 w_t y_t - 2 w_t a_t \\
= (y_t - a_t)^2 + w_t^2 + 2 w_t y_t - 2 w_t a_t
\]
and
\[
\hat{c}_{S,t} = \frac{1}{1 - \lambda} \hat{c}_t - \frac{\lambda}{1 - \lambda} \hat{c}_{H,t}
\]
thus
\[
c_{S,t}^2 = \frac{1}{(1 - \lambda)^2} \hat{c}_t^2 + \left( \frac{\lambda}{1 - \lambda} \right)^2 \hat{c}_{H,t}^2 - 2 \left( \frac{1}{1 - \lambda} \right) \left( \frac{\lambda}{1 - \lambda} \right) \hat{c}_t \hat{c}_{H,t} \\
= \frac{1}{(1 - \lambda)^2} \hat{c}_t^2 + \left( \frac{\lambda}{1 - \lambda} \right)^2 (w_t^2 + l_t^2 + 2 \hat{w}_t l_t) - \frac{2\lambda}{(1 - \lambda)^2} \hat{c}_t (\hat{w}_t + l_t) \\
= \frac{1}{(1 - \lambda)^2} \hat{c}_t^2 + \left( \frac{\lambda}{1 - \lambda} \right)^2 (\hat{w}_t^2 + \hat{y}_t^2 + a_t^2 - 2 \hat{y}_t a_t + 2 \hat{w}_t \hat{y}_t - 2 \hat{w}_t a_t) \\
- \frac{2\lambda}{(1 - \lambda)^2} (\hat{w}_t \hat{w}_t + \hat{y}_t^2 - y_t a_t)
\]
then

\[
(\lambda \hat{c}_{H,t}^2 + (1 - \lambda) \hat{c}_{S,t}^2)
\]

\[
= \lambda \left( y_t^2 + a_t^2 - 2y_t a_t + \hat{w}_t + \hat{\gamma}_t - 2w_t a_t \right) + \frac{1}{(1 - \lambda)} y_t^2 + \frac{\lambda^2}{(1 - \lambda)} \left( \hat{w}_t^2 + \hat{\gamma}_t^2 + a_t^2 - 2\hat{y}_t a_t + 2\hat{\omega}_t \hat{\gamma}_t - 2\hat{\omega}_t a_t \right) - \frac{2\lambda}{(1 - \lambda)} \left( \hat{y}_t \hat{w}_t + \hat{\gamma}_t^2 - y_t a_t \right)
\]

collecting terms

\[
(\lambda \hat{c}_{H,t}^2 + (1 - \lambda) \hat{c}_{S,t}^2)
\]

\[
= \left( \lambda + \frac{\lambda^2}{(1 - \lambda)} \right) w_t^2 + \left( \lambda + \frac{1}{1 - \lambda} + \frac{\lambda^2}{(1 - \lambda)} - \frac{2\lambda}{(1 - \lambda)} \right) y_t^2 + \left( \lambda + \frac{\lambda^2}{(1 - \lambda)} \right) a_t^2 + \left( \lambda + \frac{\lambda^2}{(1 - \lambda)} \right) \hat{y}_t a_t + 2 \left( \lambda + \frac{\lambda^2}{(1 - \lambda)} \right) \hat{w}_t^2 + \left( \lambda + \frac{\lambda^2}{(1 - \lambda)} \right) \hat{\gamma}_t^2 + \left( \lambda + \frac{\lambda^2}{(1 - \lambda)} \right) \hat{\omega}_t a_t
\]

simplifying

\[
(\lambda \hat{c}_{H,t}^2 + (1 - \lambda) \hat{c}_{S,t}^2) = \left( \frac{\lambda}{(1 - \lambda)} \right) w_t^2 + \left( \frac{\lambda}{1 - \lambda} + \frac{\lambda^2}{(1 - \lambda)} \right) y_t^2 + \left( \frac{\lambda}{1 - \lambda} \right) a_t^2 - 2 \left( \frac{\lambda}{1 - \lambda} \right) \hat{w}_t^2 + 2 \left( \frac{\lambda}{1 - \lambda} \right) \hat{\gamma}_t^2 + \left( \frac{\lambda}{1 - \lambda} \right) \hat{\omega}_t a_t
\]

Using this results and considering that \( a_t \) is independent of policy the welfare function can be rewritten as

\[
\frac{W_t - W}{uC} = \frac{1}{2} \left[ (1 - \sigma) \lambda \left( \frac{1}{(1 - \lambda)} \right) w_t^2 - (\sigma + \phi) y_t^2 - 2(1 - \sigma) \lambda \left( \frac{1}{(1 - \lambda)} \right) w_t a_t + 2\hat{w}_t \hat{\gamma}_t + 2(1 + \phi) y_t a_t \right] - (d_{w,t} + d_{p,t}) + \text{tip}
\]

Next we have to rewrite some terms. Recall that

\[
(\sigma + \phi) y_t^{Eff} = (1 + \phi) a_t + \psi_t
\]

thus

\[
(\sigma + \phi) y_t^{Eff} = (1 + \phi) y_t a_t + \psi_t
\]

Also

\[
(\sigma + \phi) \left( y_t - y_t^{Eff} \right)^2 = (\sigma + \phi) \left( y_t^2 + \left( y_t^{Eff} \right)^2 - 2y_t y_t^{Eff} \right)
\]

\[
= (\sigma + \phi) \left( y_t^2 + \left( y_t^{Eff} \right)^2 \right) - 2(\sigma + \phi) y_t y_t^{Eff}
\]
substituting for the previous result

\[(\sigma + \phi) \left(y_t - y_t^{eff}\right)^2 = (\sigma + \phi) \left(y_t^2 + \left(y_t^{eff}\right)^2 \right) - 2 (1 + \phi) y_t a_t - 2 y_t \psi_t\]

which implies that

\[2 (1 + \phi) y_t a_t + 2 y_t \psi_t = (\sigma + \phi) \left(y_t^2 + \left(y_t^{eff}\right)^2 \right) - (\sigma + \phi) (y_t - y_t^{eff})^2\]

In this case

\[\frac{W_t - W}{u_C} = \frac{1}{2} \left[ \frac{(1 - \lambda)}{(1 - \lambda)} (w_t^2 - 2 w_t a_t) - (\sigma + \phi) x_t^2 \right] - (d_{w,t} + d_{p,t}) + t i p\]

where \(x_t = (y_t - y_t^{Eff})\) and given that \(y_t^{Eff}\) is independent of policy. Also notice that

\[w_t^{eff} = a_t\]

which is a term independent of policy. Multiplying \(w_t^{Eff}\) by \(w_t\) we get:

\[w_t w_t^{eff} = w_t a_t\]

Next

\[(w_t - w_t^{eff})^2 = w_t^2 + \left(w_t^{eff}\right)^2 - 2 w_t w_t^{eff}\]

combining

\[(w_t - w_t^{eff})^2 = w_t^2 - 2 w_t a_t + \left(w_t^{eff}\right)^2\]

which implies

\[w_t^2 - 2 w_t a_t = (w_t - w_t^{eff})^2 - (w_t^{eff})^2 = \tilde{\omega}_t^2 - (w_t^{eff})^2\]

Substituting the latter into the welfare loss function and considering that \(w_t^{eff}\) is a term independent of policy, we get

\[\frac{W_t - W}{u_C} = \frac{1}{2} \left[ \frac{(1 - \lambda)}{(1 - \lambda)} \tilde{\omega}_t^2 - (\sigma + \phi) x_t^2 \right] - (d_{w,t} + d_{p,t}) + t i p\]

Using Woodford Lemma 1 and Lemma 2, we can finally write the present discounted value of the Central Bank loss function as

\[L = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[ \frac{(\sigma - 1)\lambda}{\lambda} \tilde{\omega}_t^2 + (\sigma + \phi) x_t^2 + \frac{\theta_w}{\kappa_w} (\pi_t w)^2 + \frac{\theta_p}{\kappa_p} \pi_t^2 \right] + t i p\]

Notice that if \(\sigma < 1\) deviation of the real wage from its efficient level leads to a lower society’s loss.
A.3.1 Derivation of the welfare function under flexible wages:

Remember that in the case in which wages are fully flexible, the labor supply is:

$$\omega_t = \sigma c_t + \phi d_t - \psi_t = (\sigma + \phi) y_t - \phi a_t - \psi_t - \phi d_{p,t}$$  \hspace{1cm} (63)$$

hence, subtracting the efficient equilibrium to the LHS and the RHS of the previous equation

$$\tilde{\omega}_t = (\sigma + \phi) x_t - \phi d_{p,t}$$  \hspace{1cm} (64)$$

where we use the fact that \(d_{p,t} - d_{p,t}^{Eff} = d_{p,t}\) (given that \(d_{p,t}^{Eff} = 0\)). Moreover, we know \(a_t = a_t^{Eff}\) and that \(\psi_t = \psi_t^{Eff}\) and terms multiplied by \(-\phi d_{p,t}\) are terms higher than second order. Then

$$\tilde{\omega}_t^2 = (\sigma + \phi)^2 x_t^2$$

this means that the welfare-loss can be re-written as follows:

$$L = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left( (\sigma + \phi) \left[ 1 + (\sigma - 1) (\sigma + \phi) \frac{\lambda}{1-\lambda} \right] x_t^2 + \frac{\theta_p}{\kappa_p \pi_t^2} \right)$$

Notwithstanding wage flexibility there is an additional term with respect to a fully ricardian framework, given by \(\frac{(\sigma+\phi)(\sigma-1)\lambda}{1-\lambda} x_t^2\). Two conditions are necessary for the presence of this additional term. Once again this is due to the presence of rot agents and similarly it disappears when \(\sigma = 1\). Also, when \(\sigma < 1\), the identified additional term leads to a reduction in society’s welfare loss.