Implementing Collusion by Delegating Punishment: The Role of Communication *

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Abstract

This paper studies collusion in a Bertrand duopoly with imperfect monitoring. Firms cannot directly commit to marginal cost pricing, but they can delegate the implementation of punishment to a third party, called the arbitrator. Communication arises because the arbitrator must learn the firms’ profit realizations in order to know when to start punishment. I show that the optimal collusive equilibrium is Nash implementable by a fairly simple "cartel mechanism", though not uniquely so. I also study the effect of public demand information on collusion with and without strategic delegation. I conclude that suppressing inter-firm communication is a key element in any antitrust policy designed to prevent collusion.

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1 Introduction

Competition authorities tend to be alarmed when they discover mechanisms which provide firm-level information on prices or quantities to competitors in the same industry. Two examples from EU case law are the UK Tractor Case (1992), and the Cement Case (1994). In the UK Tractor Case, the UK trade association of manufacturers and importers of agricultural machinery (Agricultural Engineers Association Ltd) collected information on tractor registration (vehicle licensing) from the Department of Transport and distributed it to its members. This information identified the retail sales and market shares of each of the eight firms on the UK market with detailed breakdowns by product, territory and time periods. The Cement Case differed from the UK Tractor Case insofar as it was price information (not sales) that was communicated by the European Cement Association (Cembureau) to its members (cement manufacturers from 19 European countries).

Finding such hard evidence on information exchange is almost requisite if the antitrust authority wants to build a solid price-fixing case. In the light of this legal practice, it is quite surprising how many cartels continue to convene frequently and to serve as platforms for information exchange even though this significantly increases the risk of exposure by law-enforcement agencies. One is left to wonder why firms opt for the risky alternative of overt collusion when they might as well collude tacitly and thereby avoid prosecution altogether?

The answer proposed in this paper starts from the observation that the collusive equilibria studied in the literature on tacit collusion are not renegotiation-proof: The threat of ”punishment” (for instance Nash reversion) needed to maintain cartel discipline has no bite if firms can expect such punishment to be bargained away whenever it is supposed to be implemented (see Farrell and Maskin (1989)). One way to achieve commitment is then to delegate punishment to an agent through an appropriately designed contract which immunizes the latter against renegotiation. While - at least nowadays - a price fixing agreement cannot be enforced in court, a mandate to a third party to charge favorably low
prices in some future period should in principle be enforceable.

The idea that firms may delegate pricing or output decisions in order to modify the competitive interaction among them is known under the name of "strategic delegation". This concept has been applied to explain the structure of vertical restraints (Rey and Stiglitz (1988, 1995), Bonanno and Vickers (1988), Gal-Or (1991)) and the internal organization of firms (Vickers (1985), Fershtman and Judd (1987)). The application of strategic delegation to collusion in a Bertrand supergame is novel to this paper.

I set up a model where such delegation of punishment to a third party is assumed to be feasible. I call this agent a "arbitrator"; the arbitrator's role is to implement marginal cost pricing when called to do so. Now, the need to communicate arises naturally because the arbitrator needs to learn the firms' profit realizations (not directly observable to the arbitrator) in order to know when to start punishment. I show that the optimal collusive equilibrium (i.e. the standard non-renegotiation-proof equilibrium) can be implemented with a fairly simple "cartel mechanism" composed of a $2 \times 2$ dimensional message space and an allocation rule that maps message profiles into probabilities of marginal cost pricing. I discuss the problem of multiplicity of equilibria at the reporting stage of the game, which implies that there is no cartel mechanism which can uniquely Nash implement the optimal collusive equilibrium.

I also study the role of noisy public demand information that becomes available to firms ex post, i.e. after their profits are realized. In the standard framework where firms can directly implement punishment, such demand information mitigates the monitoring problem and therefore affects outcomes in two ways: (i) It expands the parameter space where collusion is sustainable, and (ii) it raises the expected payoff from collusion. I show that whenever (i) applies, collusion under delegated punishment can be implemented by the same simple cartel mechanism that was sufficient in the simpler case without demand information.

The key ingredients of my model - the presence of a third party who col-
lects information with the sole purpose of implementing punishment, and the necessity to communicate even if all the relevant information is already common knowledge among the competitors - are two features that have not been studied in the collusion literature before, but seem to play an important role in practice.

A canonical example of a trade association helping to enforce a collusive agreement is the Sugar Institute analyzed by Genesove and Mullin (2001). This trade association was formed by 14 firms comprising nearly all the cane sugar refining capacity in the United States, and operated from December 1927 until 1936. Among other things, it collected information on its members’ business conduct through its own investigators, and if it found indications of cheating, provided a forum for accusation and rebuttal.

One interesting analogy to my model is that the reported information did not always coincide with the true facts:

- The accusation could be factually wrong: a concession on one barrel of caked sugar was wrongly reported as a concession on a much larger amount of powdered sugar by a Sugar Institute investigator.
- Or a firm employee or direct broker may simply have made an error in invoicing or shipping. (p. 389)

Most significantly, the punishment mechanism of the cartel differed markedly from the predictions of collusion theory in that retaliation was not immediate; instead, "the Sugar Institute served as a court", providing a mechanism "by which firms can first judge whether cheating has in fact occurred before taking action.” (p. 389) The firms also seemed eager to tap any additional source of information beyond their own profit realizations: "Market share is a noisy indicator of cheating; and with direct evidence available, the refiners evidently preferred to rely on that instead." (p. 394)

The authors argue that this approach "delayed, and perhaps restricted, retaliation against violations of the agreement” (p. 387). They quote several instances where deviators got away with what was very likely an attempt to cheat. "...Firms accept some cheating so as not to punish inappropriately.”
This corresponds nicely to my model where firms may want to trigger punishment, but fail to coordinate their messages to the arbitrator, so that the latter has no mandate to implement punishment.

A number of very insightful examples of monitoring strategies in cartels are documented by Harrington (2006). He discusses two cases where firms reported to outside individuals whose only function was to collate the information received by the cartel members: the copper plumbing tubes cartel (p. 53) and the plasterboard cartel (p. 54). In a number of instances, firms had access to information that allowed them to verify whether their rivals’ reports were correct or not. In other words, the sales information in question was actually common knowledge among the firm, and yet they subjected themselves to the reporting procedure. A striking example is the sorbates cartel:

...the group started noticing the existence of “grey material” – which corresponded to the difference between the self-declared sales results and the published Japanese export data. This “grey material” was due to the fact that the producers did not report all of their sales results to the group. The self-declared figures were not verified and the group assumed that the Japanese export data was accurate [and therefore the discrepancy was due to the producers not reporting their sales accurately]. Hoechst was always aware of the Japanese export statistics because it was a member of the Chemical Industrial Products Export Co-operative (CIPEC), a Japanese organised export cartel that had no relationship with the conspiracy, and, consequently, had access to the Japanese export statistics. (Harrington (2006), p. 52)

The paper is organized as follows: Section 2 reviews the most important contributions to collusion and information exchange. Section 3 introduces the model. Section 4 studies the properties of the optimal collusive equilibria under direct punishment. Section 5 introduces delegated punishment and characterizes
the cartel mechanism needed to implement the equilibria analyzed in Section 4. Finally, Section 6 summarizes the main findings and draws policy conclusions.

2 Related Literature

Until recently, economists have devoted little attention to the functions that cartel institutions perform for colluding firms. Kühn (2001) points out one important reason why firms may need to communicate explicitly, namely the multiplicity of collusive equilibria; when firms are not sure which of the many collusive equilibria to play, communication may help resolve this strategic uncertainty. To the extent that such coordination problems are more prevalent in cartels composed of a large number of firms and/or asymmetric firms, recent empirical evidence provided by Davies and Olczak (2008) corroborates this argument. However, this does not explain why firms need to meet so frequently, and why they would exchange information on past conduct if their intention is to coordinate on future actions.

Information on past conduct will be relevant to collusion when the rivals’ actions are not directly observable to a firm, so that deviations from the collusive agreement are harder to detect and hence to punish. Stigler (1964) was the first one to point out that without observability of rivals’ behavior, collusion will be more difficult or even impossible to sustain, unless firms can design their collusive agreement to provide the right incentives.

Green and Porter (1984) construct such a collusive agreement in a model where players observe a public outcome (namely the market-clearing price) that imperfectly signals the actions played (namely individual output decisions). They show that ”price wars” (i.e. episodes of high output and low prices) need not indicate a collapse of collusion, but can be part of the firms’ equilibrium strategies to ensure tacit collusion in the presence of stochastic demand shocks.

Note that a public signal serves two distinct purposes in such a game: first, it provides information to the agents (which could be achieved by a private signal as well), and second, it allows firms to coordinate their behavior on the
signal’s realizations (which is not the case for a (noisy) private signal, as then the state of the world will no longer be common knowledge among the agents). Therefore, there is no need for communication, even in a framework of imperfect monitoring, as long as all equilibrium strategies only condition on public information. Communication will be necessary and enhance collusion only if there is privately observed information, which must be made public to allow players to coordinate their behavior on them. (see Kandori and Matsushima (1998), Compte (1998)).

To my knowledge, Harrington and Skrzypacz (2010) is the only paper which develops a full-blown collusive mechanism inducing firms to fully disclose their unobservable actions. This mechanism consists of a reporting game, in which firms can send (true or false) messages about their privately known output levels, and a transfer scheme whereby firms who reported to have sold more than their granted share in the collusive industry output have to compensate those firms who undersold. The authors identify conditions under which a collusive equilibrium with truthful reporting exists. While this model convincingly explains the functioning of a number of recently exposed high-profile cartels like the Lysine cartel, information exchange arises only in connection with side-payments. However, not all cartels which engaged in information exchange on past outputs and prices also ran a system of side-payments. Moreover, the model does not explain how firms solve the problem of renegotiation: all collusive equilibria are sustained by the threat of Nash reversion, which is simply assumed to be implementable.

A different, but related strand of literature studies static oligopoly games in which firms may either share or conceal private information they hold. The question is whether such information exchange will arise non-cooperatively, and if so, whether or not it is profitable for firms. While each firm will always want to learn its rivals’ information, it is not clear that the firm will also find it in its interest to voluntarily disclose the information it holds itself. The incentives to share information with rival firms was formally studied by Vives (1984) in a duopoly model with differentiated products where firms have private informa-
tion about an uncertain linear demand. Vives finds that if the goods are substitutes, it is a dominant strategy for each firm to share information in Bertrand competition, while it is not under Cournot competition. Moreover, the result is reversed if the goods are complements. These findings were generalized by Raith (1996) and extended to costly information acquisition by Jansen (2008).

3 The model

Consider an infinitely repeated duopoly game where two symmetric firms produce a homogeneous good at constant marginal cost. The firms set prices every period. Buyers can perfectly observe both prices and will all buy from the low-price firm. Each firm only knows its own price but cannot observe the rival’s price. This type of imperfect monitoring is typically found in customer markets where the buyers are large firms searching the market for potential input providers.

Demand for the product is stochastic; in a given period $t$, demand will be zero with probability $\alpha$ (the "low-demand state"), and positive with probability $1 - \alpha$ (the "high-demand state"):

$$ D_t(p) = I_tD(p) $$

where

$$ I_t = \begin{cases} 0 & \text{with prob. } \alpha \\ 1 & \text{with prob. } 1 - \alpha \end{cases} $$

$D(p)$ is a standard downward sloping demand function, and realizations of $I_t$ are iid over time. Firms cannot directly observe the state of demand.

For the high-demand state, denote the per-period monopoly profits by $\Pi^m$. We assume that the two firms share the market equally whenever they charge the same price. Thus, in a period of high demand, each firm’s profit under collusion will be $\Pi^m/2$. Next period’s profits are discounted at rate $\delta < 1$.

If a firm realizes profits $\Pi^m/2$, it can perfectly infer the other firm’s behavior and vice versa. Then, it is common knowledge that demand was high and both firms set the collusive price. If instead a firm does not sell anything at some date, it does not know \textit{apriori} whether this is due to a low realization of demand or to its competitor charging a lower price. Each firm can however observe its
own profits; thus, it is always common knowledge that at least one firm realized zero profits: Either demand was low, so that the other firm realized zero profits as well, or the other firm undercut, thus stealing all profits from its rival.

In addition to their observations of own profits, let the firms receive a noisy signal on the demand realization after each period.¹ The signals are iid over time, and the conditional probabilities are characterized as follows:

$$
\sigma_l = \Pr(S_t = 0 | I_t = 0) > \frac{1}{2} \\
\sigma_h = \Pr(S_t = 1 | I_t = 1) > \frac{1}{2}
$$

If the actual state of demand was low ($I_t = 0$), the signal will indicate low demand ($S_t = 0$) with probability $\sigma_l$, and analogously for $\sigma_h$. Probabilities $\sigma_l$ and $\sigma_h$ are known to the firms, and they both exceed 1/2, i.e. the signals are informative. Signal precision may vary across demand states, i.e. it could be that $\sigma_l \neq \sigma_h$. The signal is public, so that its realization is common knowledge.

4 Collusion under optimal direct punishment

Let us start with the standard assumption that firms can implement punishment without intermediation. The equilibrium analysis for this case is analogous to the one in Amelio and Biancini (2009), just extended to account for the presence of demand signals. It turns out that this extension is non-trivial and deserves full analytical treatment.

Under the information structure imposed above, play of the infinitely repeated game generates both a private and a public information history. For each firm, the sequence of its prices and sales in each period constitutes its private history. The public history is the sequence of information which both firms observe. In our case, at each stage, it is common knowledge whether or not at least one firm realized zero profits, and whether the signal indicated high or low demand.

¹For simplicity, I assume that this signal is provided exogenously and for free. Alternatively, one could introduce an information acquisition stage into the model, where firms can decide how much to invest into signals of varying precision.
The collusive equilibrium under optimal punishment can be characterized as follows: along the collusive path, the two firms charge the collusive price until at least one firm makes zero profit. Both this event and the realization of the demand signal are now public history. Conditional on this public history, optimal punishment requires that firms coordinate on a randomization device, and either jointly stay in the collusive phase or revert to the Bertrand-Nash equilibrium forever.

Let us restrict attention to symmetric perfect public equilibria (SPPE) in pure strategies. An SPPE is a symmetric strategy profile in which players condition their actions on the public history (not on their private information) at each point in time. Following Abreu, Pearce, and Stacchetti (1986), I transform the repeated game into an equivalent static game in which the payoffs are decomposed into the sum of a stage game payoff and a continuation value.

After the realization of profits and the demand signal, in each period a public random variable is first drawn and then observed by all players. This public randomization device allows the two firms to coordinate on the punishment. Denote by $\bar{v} = 0$ the minmax of the repeated game, and by $\bar{\pi}(\delta, \alpha, \sigma_l, \sigma_h)$ the ex-ante maximal payoff. Then the set of payoffs supported by SPPE is $E_S(\delta, \alpha, \sigma_l, \sigma_h) = [\bar{v}, \bar{\pi}(\delta, \alpha, \sigma_l, \sigma_h)]$, which is compact, non-empty and convex. Moreover, an SPPE that supports the ex-ante maximal payoff $\bar{\pi}(\delta, \alpha, \sigma_l, \sigma_h)$ always exists. I focus on this optimal SPPE because it is Pareto-dominant from the point of view of the firms. This optimal equilibrium can be implemented by randomizing only between the two extremal points of the set $E_S(\delta, \alpha, \sigma_l, \sigma_h)$.

Firms start playing the monopoly price and, depending on the public information, they will stick to that strategy with a certain probability, and move to Nash reversion with the complementary probability. If both firms collude, then variations in their profit levels will fully reflect demand conditions, and so the public history observed by both firms allows them to distinguish each of the following four events:
Denoting by $\beta_s$ the probability of Nash reversion following Event $s$, the optimal collusive equilibrium (OCE) can be written as the solution to the following problem:

$$\max_{\{\beta_s\}_{s=1}^4} v = (1-\alpha) \Pi^m/2 + \delta \sum_{s=1}^4 \pi_s [(1-\beta_s)v + \beta_sv]$$

subject to:

(C1) $v \geq (1-\alpha) \Pi^m + \delta \{[(\pi_2 + \pi_3)(1-\beta_3)v + \beta_3v] + [(\pi_1 + \pi_4)(1-\beta_4)v + \beta_4v]\}$

(C2) $\{\beta_s\}_{s=1}^4 \in [0,1]$

Constraint (C1) represents the firms’ incentive compatibility constraint: collusion is sustainable if the collusive payoff, $v$, is at least as high as the payoff from cheating. If a firm cheats while the other continues to collude, it can appropriate the full monopoly profit $\Pi^m$, provided demand is high in the period when cheating occurs. In any case, the firm that was cheated on will make zero profits. With the unconditional probability that the signal indicates low demand, $\Pr(S_t=0) = \pi_2 + \pi_3$, the other firm will believe that Event 3 has occurred, so that Nash reversion will be triggered with probability $\beta_3$. Analogously, with probability $\Pr(S_t=1) = \pi_1 + \pi_4$, the public history is identical to the one that would arise after Event 4, and so Nash reversion will be triggered with probability $\beta_4$.

**Proposition 1 (OCE with signals)** If demand signals are available, there exist threshold values for $\alpha$ and $\delta$, namely $\bar{\alpha} = \frac{\sigma_h}{\sigma_h + 1-\sigma_l}$, $\alpha = \frac{1-\sigma_h}{\sigma_l + 1-\sigma_h}$, $\delta = \frac{1}{\alpha\sigma_l + (1-\alpha)(1+\sigma_h)}$, and $\hat{\delta} = \frac{1}{(1-\alpha)^2}$, such that

(i) the collusive equilibrium yielding maximal payoff $\pi(\delta, \alpha, \sigma_l, \sigma_h)$ exists for $\delta$ arbitrarily close to 1 iff $\alpha < \bar{\alpha}$;

(ii) the equilibrium strategies are as follows: firms optimally set $\beta_1^* = \beta_2^* = 0$. 

<table>
<thead>
<tr>
<th>Event $s$</th>
<th>$I_t$</th>
<th>$S_t$</th>
<th>Probability $\pi_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$(1-\alpha)\sigma_h$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>$(1-\alpha)(1-\sigma_h)$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>$\alpha\sigma_l$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>$\alpha(1-\sigma_l)$</td>
</tr>
</tbody>
</table>
(ii-a) If $\delta \in [\hat{\delta}, 1]$, then $\beta_3^* = 0$ and $\beta_4^* \in (0, 1]$. 
(ii-b) If $\alpha < \bar{\alpha}$ and $\delta \in [\hat{\delta}, \bar{\delta})$, then $\beta_3^* \in (0, 1]$ and $\beta_4^* = 1$.
(ii-c) If $\delta < \min\{\hat{\delta}, \bar{\delta}\}$, collusion is not sustainable.

**Proof:** see appendix. □

It is quite intuitive that the firms will never punish if Event 1 or 2 occurs: In both cases, each firm makes profits of $\Pi_m/2$ and can thus immediately infer that the other firm colluded as well. Punishment would only reduce the payoffs from collusion, without strengthening enforcement in any way. If instead firms make zero profits, they face an inference problem, and so punishment must be induced with some positive probability for cooperation to be sustainable.

In this case, the demand signal plays an important role because it partitions the state space further and hence allows firms to condition the probability of Nash reversion on its realization. If firms are sufficiently patient, they will skip punishment altogether if the signal indicates low demand, and punish with positive probability only if the demand signal indicates high demand. For some values of $\alpha$ and $\delta$, this strategy may not provide enough deterrence. In this case, firms may have to enter punishment with some positive probability even if the signal indicates low demand, and they will do so with certainty if the signal indicates high demand.

Note that such collusive equilibria will only exist if negative demand shocks are not too likely, i.e. if $\alpha < \bar{\alpha}$, where $\bar{\alpha} = \Pr(I_t = 1|S_t = 1)$, i.e. the firms’ Bayesian update on the state of demand following a signal realization indicating high demand. In other words, the better firms are at detecting the high-demand state, the larger the parameter space for which collusion is sustainable.

The benchmark case without signals  Consider now the same setup as before, but suppose that there are no informative signals on the state of demand available. Then, the only event that firms can condition punishment on is the
profit realization. The OCE then solves:

\[
\begin{align*}
\max_{\gamma_1, \gamma_2} v &= (1 - \alpha) \Pi^m / 2 + \delta \{ (1 - \alpha) [(1 - \gamma_1) v + \gamma_1 v] + \alpha [(1 - \gamma_2) v + \gamma_2 v] \} \\
\text{subject to:} \\
(C1a) & \quad v \geq (1 - \alpha) \Pi^m + \delta [(1 - \gamma_2) v + \gamma_2 v] \\
(C2a) & \quad \gamma_1, \gamma_2 \in [0, 1]
\end{align*}
\]  

(2)

As shown in Amelio and Biancini (2009), firms will never find it optimal to punish when they make positive profits, while punishing with positive probability whenever they realize zero profits. The solution to optimization problem (2) as derived by Amelio and Biancini (2009) is reproduced here for convenience:

**OCE without signals:**

(i) The collusive equilibrium yielding maximal payoff \( v(\delta, \alpha) \) exists for \( \delta \) arbitrarily close to 1 iff \( \alpha < \frac{1}{2} \).

(ii) The equilibrium strategies are as follows: firms never punish if they both make profits of \( \Pi^m / 2 \), i.e. they optimally set \( \gamma_1^* = 0 \). If they make zero profits, they will switch to Nash reversion with probability

\[
\gamma_2^* = \frac{1 - \delta}{\delta (1 - 2\alpha)}.
\]

(3)

We can now evaluate the impact of public demand information on the sustainability and profitability of collusion.

**Proposition 2** Comparing the OCEs solving problem (1) to those of problem (2), we have that:

(i) demand signals expand the parameter space where collusion is sustainable;

(ii) they raise the payoffs from collusion for those parameter values where collusion is sustainable with and without signals.

**Proof:** see appendix. \( \square \)
Not surprisingly, I find that firms unambiguously benefit from the availability of demand signals: they reduce the (average) probability of Nash reversion following a negative demand shock, thus raising firms’ expected payoffs compared to the scenario without signals. Moreover, if firms can fine-tune the probability of punishment to the realization of the demand signal, they will be able to collude even for rather high levels of $\alpha$ and low levels of $\delta$, i.e. in cases where demand shocks are too likely, or firms are too impatient, to sustain collusion without signals.

5 Collusion when punishment is delegated

Let us now turn to the case where firms cannot directly commit to marginal cost pricing, but can avail themselves of the services of a third party, called the arbitrator, who will implement such prices on their behalf if requested to do so. I will not characterize the structure of the contract between firms and arbitrator, but simply assume that it is in place and enforceable.

If profit and demand realizations are also observable to the arbitrator (not just the firms), then nothing changes relative to the setup studied in the last section: Upon observing a particular realization, the arbitrator will operate the appropriate randomization device and either implement marginal cost pricing or instruct the firms to continue colluding.

More realistically, however, the arbitrator may not have access to the same profit and demand information as the market participants. Then, it will be necessary for the firms to communicate this information to the arbitrator for the latter to be able to take action. Assume that in each period $t$, firms play a two-stage game, where they first set prices (as before), and then, after they realized their profits, they simultaneously send a message to the arbitrator. The arbitrator may then either implement marginal cost pricing in $t+1$, or allow firms to set prices themselves next period.

Definition: A cartel mechanism is a sequence of message spaces and mappings $\{M_1, M_2, \tilde{g}(\cdot)\}_{t=0}^{\infty}$, where the mapping $\tilde{g}(\cdot) : M_1 \times M_2 \rightarrow [0,1]$ denotes
the probability of marginal cost pricing in $t+1$ assigned to every possible pair of messages.

Of course, such communication has its own strategic implications, which we will now analyze in more detail. Let us start with the case where neither firms nor arbitrator receive any demand signals. Again, play of the infinitely repeated game generates both a private and a public information history. However, the public history, i.e. the sequence of information which all players observe, is different now, because it can only comprise those outcomes observable to the arbitrator as well. In particular, it is no longer part of the public history whether or not at least one firm realized zero profits. With delegated punishment, the public history consists of the messages sent by both firms at the end of each period, the outcome of the randomization process, and whether or not the arbitrator took action in a particular period, i.e. whether or not the arbitrator implemented marginal cost pricing. Denote this element of the public history by $A_t$ defined as follows:

$$A_t = \begin{cases} 1 & \text{if arbitrator implemented } p_t = c \\ 0 & \text{otherwise} \end{cases}$$

I will argue that the following strategies will allow firms to Nash implement the OCE solving problem (2) by a simple two-tier cartel mechanism relying only on the public history:

In $t = 0$, the game starts in the collusive regime: both firms $i = 1, 2$ set the collusive price $p_{ti} = p^M$. After profits are realized, each firm sends the message $m_i = 0$ if it realized zero profits, and $m_i = 1$ otherwise. The two messages are sent simultaneously. The four possible outcomes of the reporting game map into the following probabilities for Nash reversion:

$$\tilde{g}_c(m_1, m_2) = \begin{cases} \gamma^* & \text{if } m_1 = m_2 = 0 \\ 1 & \text{if } m_1 \neq m_2 \\ 0 & \text{if } m_1 = m_2 = 1 \end{cases}$$

where subscript $c$ stands for collusion.
If the outcome of the randomization in \( t = 0 \) calls for a shift to the punishment regime, the arbitrator will implement \( p_t = c \) in \( t = 1 \), so that \( A_1 = 1 \). Both firms report their correct profit realizations, \( m_i = 0 \), and the game remains in the punishment regime with probability

\[
\tilde{g}_p(m_1, m_2) = 1 \forall (m_1, m_2) \in M_1 \times M_2
\]

where subscript \( p \) stands for punishment.

If instead the stage game in \( t = 0 \) ended without regime shift, i.e. \( A_1 = 0 \), then both firms set the collusive price \( p_{t1} = p^M \), and play continues as in \( t = 0 \).

**Proposition 3:** The cartel mechanism characterized by \( M_1 \times M_2 = \{0, 1\} \times \{0, 1\} \forall t \) and

\[
\tilde{g}_t(m_1, m_2) = \begin{cases} 
\tilde{g}_c(m_1, m_2) & \text{if } A_t = 0 \\
\tilde{g}_p(m_1, m_2) & \text{otherwise}
\end{cases} \forall t
\]

is sufficient for the OCE solving problem (2) to be Nash implementable.

Proof: see Appendix

The crucial point here is that the mapping \( \tilde{g}_t(m_1, m_2) \) can sustain truthful reporting by the firms as a Nash equilibrium for each state of the world: If at least one firm made zero profits, then this is common knowledge among firms. In this case, if firm \( i \) reports truthfully, firm \( j \)'s best response to this is to report truthfully as well: otherwise, Nash reversion will be triggered with certainty rather than with probability \( \gamma^*_2 \leq 1 \). Likewise, if both firms made positive profits, then truthful reporting is again a best response for each firm to the other firm’s truthful reporting. If the state of the world is always revealed correctly, the arbitrator’s actions mimic the duopoly’s actions at the OCE where firms can directly implement their punishment. Then, the cartel mechanism is just as efficient at preventing defection as direct punishment.

Of course, the problem with such reporting games is that there are multiple equilibria: If, for some reason, a firm reports \( m_i = 1 \) even when this is not the true state of the world, the other firm’s best response to this is to send a false report as well. In fact, the reporting game defined above has three Nash
equilibria, independently of the true state of the world: (i) both firms reporting $m_i = 0$, (ii) both firms reporting $m_i = 1$, and (iii) both firms mixing between $m_i = 0$ and $m_i = 1$ with probability $\pi = 1/(2 - \gamma_2^*)$. This leads us to our next result, which highlights the difficulties arising from the use of a cartel mechanism.

**Proposition 4:** There is no cartel mechanism which can uniquely implement the OCE solving problem (2) in Nash equilibrium.

Proof: see Appendix

Proposition 4 highlights how crucially the result of Proposition 3 hinges on truthful reporting at the communication stage of the game. Yet, truthful reporting is a rather fragile equilibrium: Once the colluding firms are hit by a negative demand shock, they would be better off concealing this event from the arbitrator by misreporting $m_i = 1$. In this case, they could guarantee themselves a continuation of collusion next period, rather than facing the risk of Nash reversion. In other words, the Pareto-dominant Nash equilibrium in the reporting game is $m_1 = m_2 = 1$ for each state of the world. This is also the only equilibrium that survives if we allow firms to report sequentially rather than simultaneously. But as soon as profit realizations are disconnected from reporting, the mechanism loses its deterrence effect, and so defection cannot be prevented efficiently.

Finally, let us analyze how the introduction of demand information changes the cartel mechanism required to Nash implement the collusive equilibrium. Recall that the OCE with signals can be of two types: either punishment occurs only if $I_t = 0$ and $S_t = 1$, i.e. if Event 4 occurs, but not otherwise (in particular, punishment is skipped if $I_t = 0$ and $S_t = 0$). Then, firms will only have to communicate whether or not Event 4 has occurred; in other words, Event 4 maps into message $m_i = 0$, and Events 1, 2, and 3 all map into message $m_i = 1$. If $m_1 = m_2 = 0$, the arbitrator will optimally implement marginal cost pricing with probability $\beta_4^*$, otherwise, collusion continues next period with certainty.
Define the corresponding allocation rule as:

$$\tilde{g}_c^t (m_1, m_2) = \begin{cases} 
\beta_4^* & \text{if } m_1 = m_2 = 0 \\
1 & \text{if } m_1 \neq m_2 \\
0 & \text{if } m_1 = m_2 = 1
\end{cases}$$

If instead punishment occurs with positive probability after either Event 4 or 3, then the message space must be enlarged to accommodate the third alternative. In this case, let Event 4 map into message $m_i = 0$, Event 3 into $m_i = 1$, and Events 1 and 2 both map into message $m_i = 2$. If $m_1 = m_2 = 1$, the arbitrator will optimally implement marginal cost pricing with probability $\beta_3^* > 0$; if instead $m_1 = m_2 = 0$ or the reports diverge, the arbitrator will optimally implement marginal cost pricing with probability $\beta_4^* = 1$; and if $m_1 = m_2 = 2$, collusion continues next period with certainty. Define the corresponding allocation rule as:

$$\tilde{g}_c^0 (m_1, m_2) = \begin{cases} 
\beta_3^* & \text{if } m_1 = m_2 = 1 \\
1 & \text{if } m_1 \neq m_2 \text{ or if } m_1 = m_2 = 0 \\
0 & \text{if } m_1 = m_2 = 2
\end{cases}$$

We can now state our final result regarding Nash implementability of the collusive equilibrium:

**Proposition 5:** Suppose firms (but not the arbitrator) can observe demand signals in addition to profit realizations. Then, the following cartel mechanisms are sufficient for the OCE solving problem (1) to be Nash implementable:

(i) If $\delta \in [\bar{\delta}, 1)$, then $M = \{0, 1\} \times \{0, 1\}$ and

$$\tilde{g}_c^t (m_1, m_2) = \begin{cases} 
\tilde{g}_c^t (m_1, m_2) & \text{if } A_t = 0 \\
\tilde{g}_p (m_1, m_2) & \text{otherwise}
\end{cases}$$

(ii) If $\delta \in (\bar{\delta}, \bar{\delta})$, then $M = \{0, 1, 2\} \times \{0, 1, 2\}$ and

$$\tilde{g}_c^0 (m_1, m_2) = \begin{cases} 
\tilde{g}_c^0 (m_1, m_2) & \text{if } A_t = 0 \\
\tilde{g}_p (m_1, m_2) & \text{otherwise}
\end{cases}$$

Proof: see Appendix

Interestingly, we have that whenever signals expand the parameter space where collusion can be sustained (i.e. whenever $\bar{\delta} < \bar{\delta}$), the cartel mechanism
required to implement the corresponding collusive equilibrium is as simple as the one needed to implement the OCE without signals. In particular, although the dimension of the state space increases from 2 to $2 \times 2$, a $2 \times 2$ dimensional message space is still sufficient to translate all states of the world into efficient punishment decisions. In other words, the miscoordination problem present in the reporting game is not aggravated in any way by the introduction of demand signals.

Only where $\delta \in [\bar{\delta}, \check{\delta})$, i.e. where signals do nothing to expand the parameter space where collusion is sustainable, will the message space have to be enlarged to implement the OCE.

6 Conclusion

The purpose of this paper was to study the scope for collusion when firms cannot implement punishment directly, but can avail themselves of a third party (the "arbiter") that implements such punishment for them. If firms engage in this type of strategic delegation to enforce collusion, the need to communicate arises naturally because the arbiter needs to learn the firms' profit realizations and other relevant information so as to know when to take action.

My results contribute to the discussion whether suppressing communication among horizontal competitors is an efficient policy to prevent collusion. To most people, this would seem like a straightforward and hardly debatable proposition. However, starting with McCutcheon (1997), the economics profession has been concerned that hampering communication between colluding firms may actually achieve the opposite effect, namely facilitate collusion. One reason why firms may want to communicate is to renegotiate the collusive agreement. Now, if firms can renegotiate after a defection has occurred, they will want to let bygones be bygones and relaunch collusion. This will of course soften the threat of punishment to the point where it has no bite anymore, so that defection can no longer be prevented. The optimal policy response would then be to interfere as little as possible with inter-firm communication so as to minimize the cost
of renegotiation, because this renegotiation will eventually be self-defeating for the cartel.

My results show that allowing for communication between firms is not necessarily at odds with sustaining collusive equilibria, but rather the opposite. Communication is essential whenever firms delegate punishment to a third party rather than implementing it themselves: Delegation avoids the problem of renegotiation, and communication makes sure that the arbitrator’s actions replicate exactly the strategy profile required to sustain the OCE.

My results also demonstrate that delegation comes at a cost for firms, namely the problem of sustaining truthful reporting in each period. The multiplicity of equilibria at the reporting stage implies a fair amount of strategic uncertainty for firms, so that successful collusion is far from guaranteed. The coordination problems of the Sugar Institute mentioned in the Introduction nicely illustrate this point. Therefore, firms are likely to resort to such a cartel mechanism only when direct implementation of punishment is not feasible for some reason. A tough stance on inter-firm communication will therefore allow policy makers to target those firms which have to rely on overt collusion, i.e. precisely on those cartels which cannot easily dodge antitrust prosecution by retreating to tacit collusion.

Finally, this paper shows that any additional demand information that becomes available will facilitate collusion in quite the same way under delegated punishment as under direct punishment. Thus, the key insight that improving transparency in an industry characterized by imperfect monitoring is likely to lead to higher prices remains valid. The literature provides a number of interesting examples for this effect, among them the US railroad grain rates in the 1980’s (see Fuller et al. (1990)) and the Danish ready-mixed concrete market in the early 1990’s (see Albæk et al. (1997)). The striking feature of these two examples is that the relevant market information was not provided by a cartel or trade association, but by government agencies, who were certainly hoping to achieve the opposite effect. Such practices should therefore remain taboo.
References


Appendix

Proof of Proposition 1: We start with part (ii) of Proposition 1. Recall that \( v = 0 \), and rearrange the objective function, (1), to read

\[
v_{OF} = \frac{(1 - \alpha) \Pi^m / 2}{1 - \delta \sum_{s=1}^{4} \pi_s (1 - \beta_s)}
\]

(the subscript OF will help us to distinguish the objective function from the incentive constraint, denoted by IC).

The value of collusion, \( v \), is strictly decreasing in \( \beta_1 \) and \( \beta_2 \):

\[
\frac{\partial v_{OF}}{\partial \beta_s} = - \frac{(1 - \alpha) \Pi^m / 2}{1 - \delta \sum_{s=1}^{4} \pi_s (1 - \beta_s)} \delta \pi_s < 0 \text{ for } \beta_s \in \{\beta_s\}_{s=1}^{4}
\]

Similar to the objective function, rearrange the incentive constraint (C1) to read

\[
v \geq \frac{(1 - \alpha) \Pi^m}{1 - \delta \{ (\pi_2 + \pi_3)(1 - \beta_3) + (\pi_1 + \pi_4)(1 - \beta_4) \}}
\]

We see that \( \beta_1 \) and \( \beta_2 \) enter the incentive constraint only on the left-hand side, through \( v \), while they do not affect the value of defection, i.e. the right-hand side of (C1). Hence, reducing \( \beta_1 \) and \( \beta_2 \) both raises the value of the objective function and relaxes the incentive constraint. Therefore, it must be optimal to set \( \beta_1 \) and \( \beta_2 \) to their lowest possible value, i.e. \( \beta_1^* = \beta_2^* = 0 \).
Let us now solve for the optimal $\beta_3^*$ and $\beta_4^*$.

**Part (ii-a):** Setting $\beta_1^* = \beta_2^* = 0$ and inserting for $\{\pi_s\}_{s=1}^4$, the objective function simplifies to

$$v_{OF} = \frac{(1 - \alpha) \Pi^m / 2}{1 - \delta (1 - \alpha) - \delta \alpha \sigma_l (1 - \beta_3) - \delta \alpha (1 - \sigma_l) (1 - \beta_4)}$$

Given that $\frac{\partial v_{IC}}{\partial \beta_s} < 0$ for $\beta_s \in \{\beta_3, \beta_4\}$, the incentive constraint will be binding under any solution of our maximization problem. Rewrite the incentive constraint (C1) as

$$v_{IC} = \frac{(1 - \alpha) \Pi^m}{1 - \delta \left\{(1 - \alpha) (1 - \sigma_h) + \alpha \sigma_l (1 - \beta_3) + [(1 - \alpha) \sigma_h + \alpha (1 - \sigma_l)] (1 - \beta_4)\right\}}$$

Now, take the total differential of the incentive constraint:

$$dv_{IC} = \frac{\partial v_{IC}}{\partial \beta_3} d\beta_3 + \frac{\partial v_{IC}}{\partial \beta_4} d\beta_4 = 0$$

to solve for the marginal rate of substitution between $\beta_3$ and $\beta_4$:

$$\frac{d\beta_3}{d\beta_4} = -\frac{\frac{\partial v_{IC}}{\partial \beta_4}}{\frac{\partial v_{IC}}{\partial \beta_3}} = -\frac{(1 - \alpha) \sigma_h + \alpha (1 - \sigma_l)}{(1 - \alpha) (1 - \sigma_h) + \alpha \sigma_l} < 0$$

Next, evaluate the total change in the objective function when $\beta_3$ and $\beta_4$ are traded off against each other according to this marginal rate of substitution:

$$dv_{OF} = \frac{\partial v_{OF}}{\partial \beta_3} d\beta_3 + \frac{\partial v_{OF}}{\partial \beta_4} d\beta_4$$

$$= \frac{\partial v_{OF}}{\partial \beta_3} \left\{(1 - \alpha) \sigma_h + \alpha (1 - \sigma_l)\right\} + \frac{\partial v_{OF}}{\partial \beta_4} d\beta_4$$

$$= \frac{(1 - \alpha) \Pi^m / 2}{[\mathcal{D}]^2} \delta \sigma l d\beta_4 \left[\frac{\sigma_l (1 - \alpha) \sigma_h + \alpha (1 - \sigma_l)}{(1 - \alpha) (1 - \sigma_h) + \alpha \sigma_l} - (1 - \sigma_l)\right]$$

where $\mathcal{D}$ denotes the denominator of $v_{OF}$. Now, our assumptions on parameters imply that

$$\frac{(1 - \alpha) \Pi^m / 2}{[\mathcal{D}]^2} \delta \sigma l > 0$$

and that

$$\frac{\sigma_l (1 - \alpha) \sigma_h + \alpha (1 - \sigma_l)}{(1 - \alpha) (1 - \sigma_h) + \alpha \sigma_l} - (1 - \sigma_l) > 0$$
(the latter reduces to \((1 - \alpha) (\sigma_h + \sigma_l - 1) > 0\), which is indeed satisfied by our assumption that the signals are informative, \(\sigma_l > \frac{1}{2}\) and \(\sigma_h > \frac{1}{2}\).)

Hence, we can conclude that

\[
sign \left( dv_{OF} \right) = sign \left( d\beta_4 \right)
\]
i.e. a reduction in \(\beta_3\), matched by an increase in \(\beta_4\) just sufficient for the incentive constraint to remain binding, will unambiguously increase the value of the objective function. Therefore, it must be optimal to set \(\beta_3\) to its lowest possible value, i.e. \(\beta_3^* = 0\).

Next, find the solution for \(\beta_4^*\). Insert \(\beta_3^* = 0\) into \(v_{OF}\) and \(v_{IC}\), and equate the two to solve for \(\beta_4^*\) as

\[
\beta_4^* = \frac{1 - \delta}{\delta [(1 - \alpha) \sigma_h - \alpha (1 - \sigma_l)]}
\]

(4)

For the expression above to be a valid solution, we must have \(\beta_4^* \in [0,1]\).

\(\beta_4^* \leq 1\) can be rearranged to read

\[
\delta \geq \frac{1}{\alpha \sigma_l + (1 - \alpha) (1 + \sigma_h)} = \bar{\delta}
\]

which is the condition identifying Part (ii-a) of Proposition 1.

Next, note that \(\delta \in (0,1)\) implies \((1 - \delta) / \delta > 0\). Thus, for \(\beta_4^* > 0\) to be satisfied, we must have that

\[
(1 - \alpha) \sigma_h - \alpha (1 - \sigma_l) > 0
\]

This expression can be rearranged to read

\[
\alpha < \frac{\sigma_h}{1 + \sigma_h - \sigma_l} = \bar{\alpha}
\]

which must hold by part (i) of Proposition 1.

**Part (ii-b):** Suppose \(\delta < \bar{\delta}\). This means that constraint (C2), \(\beta_4^* \leq 1\), is violated under the optimal solution derived in Part (ii-a). Let us therefore explore the solution where \(\beta_4^* \leq 1\) is binding and \(\beta_3^* > 0\). Inserting \(\beta_4^* = 1\) into \(v_{OF}\) and \(v_{IC}\), and equating the two to solve for \(\beta_3^*\) yields

\[
\beta_3^* = \frac{1 - \delta [\alpha \sigma_l + (1 - \alpha) (1 + \sigma_h)]}{\delta [(1 - \alpha) (1 - \sigma_h) - \alpha \sigma_l]}
\]

(5)
For the expression above to be a valid solution, we must have $\beta^*_3 \in [0, 1]$. To have $\beta^*_3 \leq 1$, we need
\[ \delta \geq \frac{1}{2(1 - \alpha)} = \hat{\delta} \]
For $\delta \geq \hat{\delta}$ to be compatible with $\delta < \hat{\delta}$, we must have $\hat{\delta} < \tilde{\delta}$. This condition is satisfied whenever
\[ \alpha < \frac{1 - \sigma_h}{\sigma_l + 1 - \sigma_h} = \bar{\alpha} \]
as claimed in Part (ii-b) of Proposition 1.

Finally, note that $\delta < \hat{\delta}$ implies that the numerator of $\beta^*_3$ is strictly positive, so that $\beta^*_3 > 0$ requires $(1 - \alpha)(1 - \sigma_h) - \alpha \sigma_l > 0$. This condition reduces again to $\alpha < \bar{\alpha}$, thus completing the proof of Part (ii-b).

**Part (ii-c):** If $\hat{\delta} < \tilde{\delta}$, the lowest value of $\delta$ compatible with collusion is $\hat{\delta}$ (see Proof of Part (ii-b)). If instead $\hat{\delta} \geq \tilde{\delta}$, the solution derived in Part (ii-b) does not exist, and so the solution of Part (ii-a) applies. In this case, as shown in the Proof of Part (ii-a), the lowest value of $\delta$ compatible with collusion is $\tilde{\delta}$.

**Part (i) of Proposition 1:** If $\delta$ can be arbitrarily close to 1, the solution of Part (ii-a) will apply. This solution will exist if $\tilde{\delta} < 1$, which is equivalent to
\[ \alpha < \frac{\sigma_h}{\sigma_h + 1 - \sigma_l} = \bar{\alpha} \]
This concludes the proof of Proposition 1. □

**Proof of Proposition 2:** Part (i): For $\delta$ arbitrarily close to 1, the OCE exists iff $\alpha < \bar{\alpha}$ when there are signals. The corresponding existence condition absent signals reads $\alpha < \frac{1}{2}$. By our parameter assumptions, we have that $\sigma_h > 1 - \sigma_l$, which implies $\bar{\alpha} > \frac{1}{2}$. Thus, signals expand the range of values of $\alpha$ compatible with collusion by the interval $[\frac{1}{2}, \bar{\alpha}]$.

The lower bound on $\delta$ for which the OCE exists under signals is $\min \{\hat{\delta}, \tilde{\delta}\}$. For the case without signals, using expression (3), the constraint $\gamma^*_2 \leq 1$ implies $\delta \geq \frac{1}{2(1 - \alpha)} = \hat{\delta}$, the same threshold value as defined in Proposition 1. As shown
in Part (ii-b) of Proposition 1, \( \alpha < \underline{\alpha} \) implies \( \inf \{ \underline{\delta}, \bar{\delta} \} = \underline{\delta} \), and so the range of values of \( \delta \) compatible with collusion is the same with or without signals. If instead \( \alpha \geq \underline{\alpha} \), we have that \( \inf \{ \underline{\delta}, \bar{\delta} \} = \bar{\delta} \), which means that signals expand the range of values of \( \delta \) compatible with collusion by the interval \( [\bar{\delta}, \underline{\delta}] \).

Part (ii): Comparing the value function under the OCE with signals:

\[
\tau(\delta, \alpha, \sigma_l, \sigma_h) = (1 - \alpha) \frac{\Pi_m/2}{1 - \delta (1 - \alpha) - \delta \alpha (1 - \beta_3^*) + (1 - \sigma_l)(1 - \beta_4^*)} + \sigma_l (1 - \beta_3^*) + (1 - \sigma_l)(1 - \beta_4^*) 
\]

we see that \( \tau(\delta, \alpha, \sigma_l, \sigma_h) \geq \tau(\delta, \alpha) \) if and only if \( \sigma_l (1 - \beta_3^*) + (1 - \sigma_l)(1 - \beta_4^*) \geq 1 - \gamma_2^* \), i.e. if the probability of avoiding Nash reversion after a negative demand shock is higher under signals than absent signals.

To verify that this inequality is indeed satisfied, we have to consider each of the two cases identified by Proposition 1. Let us start with the equilibrium strategies defined in Part (ii-a) of Proposition 1, and compare them to \( \gamma_2^* \) as given by expression (3). If \( \delta \in [\bar{\delta}, 1) \), then \( \beta_3^* = 0 \) and \( \beta_4^* > 0 \) is given by expression (4). Note that \( \beta_4^* \leq \gamma_2^* \) if \( \alpha \geq \underline{\alpha} \), which must hold if \( \beta_3^* = 0 \) is optimal. From this and \( \sigma_l \in (\frac{1}{2}, 1) \), it follows immediately that \( \sigma_l \cdot 1 + (1 - \sigma_l)(1 - \beta_4^*) \geq 1 - \gamma_2^* \).

Next, let us turn to the equilibrium strategies defined in Part (ii-b) of Proposition 1. If \( \alpha < \underline{\alpha} \) and \( \delta \in [\underline{\delta}, \bar{\delta}) \), then \( \beta_3^* > 0 \) as given by expression (5) and \( \beta_4^* = 1 \). Now, we have that \( \beta_3^* \leq \gamma_2^* \leq \beta_4^* = 1 \), where the first inequality follows from \( \delta \geq \underline{\delta} \). Inserting expression (5) into \( \sigma_l (1 - \beta_3^*) \geq 1 - \gamma_2^* \) and simplifying the inequality, it reduces again to \( \delta \geq \underline{\delta} \).

Thus, as claimed in Proposition 2 (ii), we have that \( \tau(\delta, \alpha, \sigma_l, \sigma_h) \geq \tau(\delta, \alpha) \), i.e. signals make collusion more profitable where it is sustainable with and without signals.\( \Box \)
Proof of Proposition 3: The OCE is Nash implementable by a cartel mechanism \( (M, \tilde{g}(\cdot)) \) if, for each state of the world \( I_t \) and for each sequence of price pairs \( H_t = \{p_{1\tau}, p_{2\tau}\}_{\tau=0}^{t} \), there exists a Nash equilibrium of the reporting game, \( (m_1^\ast (I_t, H_t), m_2^\ast (I_t, H_t)) \) such that \( \tilde{g}_t (m_1^\ast (I_t, H_t), m_2^\ast (I_t, H_t)) = \gamma_t^\ast (I_t, H_t) \) for all \( I_t \), where

\[
\gamma_t^\ast (I_t, H_t) = \begin{cases} 
\gamma_2^\ast & \text{if } I_t = 0 \text{ and } p_{i\tau} = p^M \forall \tau \leq t, i = 1, 2 \\
0 & \text{if } I_t = 1 \text{ and } p_{i\tau} = p^M \forall \tau \leq t, i = 1, 2 \\
1 & \text{otherwise}
\end{cases}
\]

is the optimal probability of Nash reversion solving problem (2).

We will now show that the cartel mechanism characterized by \( M = \{0, 1\} \times \{0, 1\} \) and \( \tilde{g}_t (m_1, m_2) = \begin{cases} 
\tilde{g}_c (m_1, m_2) & \text{if } A_t = 0 \\
\tilde{g}_p (m_1, m_2) & \text{otherwise}
\end{cases} \forall t \) satisfies this condition.

First, consider allocation rule \( \tilde{g}_p (\cdot) \). If \( \tilde{g}_p (\cdot) \) applies, the arbitrator will implement marginal cost pricing next period, and firms make zero profits. Since the game will stay under allocation rule \( \tilde{g}_p (\cdot) \) with probability 1 for any message pair \( (m_1, m_2) \in M \), firms truthfully reporting \( m_1 = 0 \) is a Nash equilibrium of the reporting game. The associated continuation payoff is therefore \( v_2 = 0 \).

Next, consider period \( t = 0 \), or any period \( t > 0 \) s.t. allocation rule \( \tilde{g}_c (m_1, m_2) \) applied up until \( t \). The following table shows the normal form representation of the reporting game defined by message space \( M = \{0, 1\} \times \{0, 1\} \) and allocation rule \( \tilde{g}_c (m_1, m_2) \):

<table>
<thead>
<tr>
<th>Firm 1’s Report</th>
<th>Firm 2’s Report</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 = 0 )</td>
<td>( m_2 = 0 )</td>
</tr>
<tr>
<td>( m_1 = 1 )</td>
<td>( m_2 = 1 )</td>
</tr>
</tbody>
</table>

If firms report \( m_1 = m_2 = 1 \), then allocation rule \( \tilde{g}_c (m_1, m_2) \) applies next period as well. Let \( V > 0 \) denote the continuation payoff under \( \tilde{g}_c (m_1, m_2) \).

If instead the two firms make diverging reports, i.e. \( m_1 \neq m_2 \), then the game
switches from $\tilde{g}_c(m_1, m_2)$ to $\tilde{g}_p(m_1, m_2)$, so that the continuation payoff is $v = 0$. Finally, if firms report $m_1 = m_2 = 0$, then the game will switch to $\tilde{g}_p(m_1, m_2)$ with probability $\gamma_2^*$, and remain under allocation rule $\tilde{g}_c(m_1, m_2)$ with probability $1 - \gamma_2^*$. The corresponding continuation value in this case is therefore $(1 - \gamma_2^*)\delta V$.

The reporting game under $\tilde{g}_c(m_1, m_2)$ has three Nash equilibria: (i) $m_1^* = m_2^* = 0$; (ii) $m_1^* = m_2^* = 1$; (iii) both firms mix between $m_i = 0$ and $m_i = 1$, playing $m_i = 0$ with probability $\pi = 1/(2 - \gamma_2^*)$. Note that these three Nash equilibria exist independently of the demand state $I_t$ and the history $H_t$. Thus, truthful reporting can always be sustained as a Nash equilibrium: if, in any given $t = 0, 1, 2, \ldots$, at least one of the two firms made zero profits, both firms reporting $m_1 = m_2 = 0$ is a Nash equilibrium; likewise, if both firms made positive profits, both firms reporting $m_1 = m_2 = 1$ is a Nash equilibrium.

If firms report truthfully in each period $t = 0, 1, 2, \ldots$, then the cartel mechanism is isomorphic to the infinitely repeated game of problem (2): In the latter, Nash reversion occurs with the exact same probabilities as in the former. The incentives to deviate from monopoly pricing are therefore identical to those of problem (2) as well. It follows that the pricing strategy of setting $p_t = p^M$ as long as the arbitrator does not implement marginal cost pricing yields an expected payoff of $V = \tau(\delta, \alpha)$, which completes the proof.\)

**Proof of Proposition 4:** The OCE solving problem (2) is uniquely implementable in Nash equilibrium by the mechanism $\{M, \tilde{g}(\cdot)\}_{t=0}^{\infty}$ if the mechanism has a unique Nash equilibrium for each $(I_t, H_t)$ and it induces the optimal Nash reversion probabilities $\gamma_t^*(I_t, H_t)$. First, note that any Nash equilibrium of the stage game, repeated infinitely often, is also a Nash equilibrium of the supergame. Thus, if the stage game has multiple equilibria, so thus the supergame. Let us therefore focus on the stage game played in period $t$.

By the Revelation Principle, any allocation that is implemented in Nash equilibrium by a mechanism $(M, \tilde{g}(\cdot))$ can also be implemented in Nash equilib-
rium by a truthful direct revelation mechanism. We can therefore limit attention to the message space \( M = \{0, 1\} \times \{0, 1\} \).

For truthful reporting to be the unique Nash equilibrium of the reporting game for each \((I_t, H_t)\), it must be the case that \(m_i = 0\) is a dominant strategy for each firm \(i = 1, 2\) whenever \(I_t = 0\), and \(m_i = 1\) is a dominant strategy for each firm \(i = 1, 2\) whenever \(I_t = 1\). But this can only be the case if \(V (I_t, H_t)\) varies with \(I_t\), i.e. if the continuation value of the game depends on the current realization of demand. For instance, under allocation rule \(\tilde{g}_c (m_1, m_2)\), whenever \(I_t = 0\), it must be the case that the payoff from \(\tilde{g}_c (0, 1)\) exceeds \(\delta V (0, H_t)\), while \(\delta V (1, H_t)\) must exceed the payoff from \(\tilde{g}_c (0, 1)\) whenever \(I_t = 1\). With \(I_t\) being iid over time, we have that \(\pi (\delta, \alpha)\) is independent of the current demand realization. Thus, if \(V (0, H_t) \neq V (1, H_t)\), we have that \(V (I_t, H_t) \neq \pi (\delta, \alpha)\) for some \(I_t\), i.e. the mechanism does not implement the OCE, but some other allocation. If instead \(V (I_t, H_t) = \pi (\delta, \alpha)\) for all \(I_t\), then truthful reporting cannot be a unique Nash equilibrium.

Let us construct another Nash equilibrium, one that does not entail truthful reporting: Suppose both firms report \(m_i = 1\) independently of their actual profit realizations. For the price setting stage of the game, this means that any defection from monopoly pricing will remain without consequences, so that collusive pricing is not sustainable. Thus, firms will play the one-shot Nash equilibrium of marginal cost pricing, followed by reports \(m_i = 1\). The game will never leave the collusive regime, i.e. we have \(A_t = 0\forall t\); the same play can therefore be repeated in any following period. Thus, we constructed another Nash equilibrium of the supergame, implemented by the same cartel mechanism as the OCE, which shows that the OCE is not uniquely implementable. 

**Proof of Proposition 5:** Part (i): In Proposition 1, we showed that if demand signals are available and \(\delta \in [\hat{\delta}, 1)\), then \(\beta_1^* = \beta_2^* = \beta_3^* = 0\) and \(\beta_4^* \in (0, 1]\) characterize the OCE. Consider a cartel mechanism where firms report \(m_i = 0\) whenever Event 4 has occurred, and \(m_i = 1\) otherwise. If \(\tilde{g}_c (m_1, m_2)\) applies,
then \( m_1 = m_2 = 0 \) is followed by a regime shift with probability \( \beta^*_1 \), while \( m_1 = m_2 = 1 \) does not induce any switch from the collusive regime. The structure of the game is exactly identical to that of Proposition 3, just that \( \gamma^*_2 \) is now replaced by \( \beta^*_4 \). It follows from Proposition 3 that truthful reporting can be sustained as a Nash equilibrium of the reporting game for every \((I_t, H_t)\), and that the cartel mechanism \((M, \tilde{g}'_t(m_1, m_2))\) can therefore exactly replicate the OCE payoffs.

Part (ii): By Part (ii-b) of Proposition 1, if \( \delta \in \left[ \underline{\delta}, \bar{\delta} \right) \), then the OCE is characterized by \( \beta^*_1 = \beta^*_2 = 0 \), and \( \beta^*_3 \in (0,1] \) and \( \beta^*_4 = 1 \). In the razor edge case where \( \delta = \underline{\delta} \), we have that \( \beta^*_3 = \beta^*_4 = 1 \), which coincides with the OCE without signals. If instead \( \delta > \bar{\delta} \), then \( \beta^*_3 < 1 \), so that there are three (not two) relevant states of the world that must be revealed to the arbitrator. Therefore, each firm must be able to send one of three different messages, i.e. the minimum dimension of \( M_i \) is three. The rest of the proof follows again from Proposition 3. \( \square \)