Abstract

We develop and estimate a structural model of informational herding in financial markets. In the model, a sequence of traders exchanges an asset with a market maker. Herd behavior, i.e., the choice to follow the actions of one’s predecessors, can arise as the outcome of a rational choice because there are multiple sources of asymmetric information in the economy. We estimate the model using transaction data on a NYSE stock in the first semester of 1994. We are able to detect the periods of the trading day in which traders herd, and find that they account for 20% of trading periods. Moreover, we find that in more than 20% of days, herding accounts for more than 50% of all trading activity. Finally, by simulating the model, we estimate the informational inefficiency generated by herding. On average, because of herding, the actual price is 0.4% distant from the full information price. Moreover, in 4.3% of trading periods, the distance between actual and full information prices is larger than 5%. This suggests that the informational inefficiency caused by herding, although not extremely large on average, is very significant in certain days.
1 Introduction

In recent years there has been much interest in herd behavior in financial markets. Especially after the financial crises of the 1990s, many scholars have suggested that herd behavior may be a reason for excess price volatility and financial systems fragility. This interest has led researchers to look for both theoretical explanations and empirical evidence.\(^1\)

The first theoretical work on herd behavior dates from the beginning of the 90’s with the seminal papers of Banerjee (1992), Bikhchandani et al. (1992), and Welch (1992). These papers do not discuss herd behavior in financial markets, but in an abstract environment, in which agents with private information make their decisions in sequence. They show that, after a finite number of agents have chosen the same action, all following agents will disregard their own private information and imitate that action. More recently, a number of papers (see, e.g., Avery and Zemsky, 1998, Lee, 1998, Cipriani and Guarino, 2008a, Cipriani and Guarino, 2008c, Park and Sabourian, 2006) have focused on herd behavior in financial markets. In particular, all these studies analyze a market where agents sequentially trade a security of unknown value. The price of the security is efficiently set by a market maker according to the order flow. The presence of a price mechanism makes herding more difficult to arise. Still there are cases in which it occurs. In Avery and Zemsky (1998) people can herd when there is uncertainty not only on the value of the asset but also on other parameters of the model. In Park and Sabourian (2006), herding arises when the structure of the signal is such that agents believe that extreme outcomes are more likely events than intermediate ones. In Cipriani and Guarino (2008a) agents herd because they have other reasons to trade, in addition to informational motives. In this model, not only agents herd, but a complete informational cascade arises.\(^2\)

Whereas the theoretical research has tried to identify the mechanisms through which herd behavior can arise, the empirical literature has followed a different track. The existing work (see, e.g., Lakonishok et al., 1992, Grin-

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\(^1\)In this paper we only study informational herding. Therefore, we do not discuss herd behavior due to reputational concerns or payoff externalities. For critical surveys of the literature on herd behavior see Gale (1996), Hirshleifer and Teoh (2003), and Chamley (2004).

\(^2\)We will define the concept of herd behavior formally later in the paper. Here we note that herd behavior refers to conformity in actions (e.g., all traders buy); informational cascade refers to the actions being completely uninformative to the other agents.
blatt et al., 1995, Wermers, 1999, and the other papers cited in the survey of Hirshleifer and Teoh, 2003) does not test these models directly, but analyzes the presence of herding in financial markets through statistical measures of clustering. These papers find that in some markets fund managers tend to cluster their investment decisions more than if they acted independently. The existing empirical research on herding is important, as it sheds light on the behavior of financial market participants and in particular on whether they act in a coordinated fashion. As the authors themselves emphasize, however, decision clustering may or may not be due to herding (for instance, it may be the result of a common reaction to public announcements). These papers cannot distinguish spurious herding from true herd behavior, i.e., the decision to disregard one’s private information to follow the behavior of others (see Bikhchandani and Sharma, 2000, and Welch, 2000). Testing informational models of herd behavior is a difficult task. In these models herding in the financial market consists in trading independently of private information. The problem that empiricists face in the task of detecting herding is that there are no data on the private information available to the traders and, therefore, it is difficult to understand whether traders make similar decisions because they disregard their own information and imitate or because of other reasons.

The purpose of this paper is to overcome this problem and offer an empirical analysis of herd behavior which is not purely statistical. We present a theoretical model of herding and estimate it using financial market transaction data. We are able to identify the periods in the trading day in which traders act as herders according to the model. This is the first empirical paper on informational herding that, instead of using a statistical, a-theoretical approach, estimates a theoretical model.\textsuperscript{3}

Our theoretical analysis is inspired by the work of Avery and Zemsky (1998), who use a sequential trading model à la Glosten and Milgrom (1985) to show the conditions under which herding can arise in financial markets. In their model, traders trade an asset of unknown value with a market maker. Traders receive private information on it. The market maker is uninformed and sets the price of the asset on the basis of the buy and sell orders that he receives. Avery and Zemsky show that, if the private information only concerns the asset fundamental value, traders will always find it optimal to

\textsuperscript{3}While there are no direct empirical tests of herding models, there is experimental work that tests these models in the laboratory: see Cipriani and Guarino (2005) and Drehman et al (2005).
trade on the difference between their own information (the history of trades and the private signal) and the commonly available information (the history only). Therefore, it will never be the case that agents neglect their information and imitate previous traders’ decisions. They also show, however, that when are multiple sources of asymmetric information between the traders and the market maker (i.e., asymmetric information not only on the asset value but also on other model parameters) herd behavior arises.

In our model, herding arises for a mechanism similar to that exposed by Avery and Zemsky. Whereas Avery and Zemsky were interested in providing some theoretical examples in which herd behavior can arise, our aim is to estimate the importance of herding in real financial market. For this purpose, we build a financial market model of herding that can be estimated using market data. An asset is traded over many days. At the beginning of each day, an informational event can occur or not. In the former case, the fundamental asset value changes with respect to the value of the previous day. It can be higher (in the case of a good informational event) or lower (in the case of a bad informational event) than the value in the previous day. In the latter case, instead, it remains unchanged. If an event has occurred, some agents will receive private information on the new asset value. These agents will trade the asset to exploit their informational advantage on the market maker. On the contrary, if no event has occurred, all traders in the market will be uninformed: they will trade for non-information reasons only, e.g., liquidity or hedging motives. While the informed traders know that they are in a market in which there is private information (since they themselves are informed) the market maker does not know whether he is in an informed or uninformed market for that day. This asymmetry in information determines a different way of updating the beliefs on the asset value by the traders and the market maker. The market maker will move the price “slowly” since he has to take into account the possibility that the asset value has not changed, the market is uninformed and all orders are coming from uninformed (noise) traders. His interpretation of the history of trades will be different from the traders’. There can be times in which, irrespective of his private signal, a trader will value the asset more than the market maker and, therefore, will find it optimal to buy; or he will value the asset less than the market maker and will find it optimal to sell. These are the periods when herd buy or herd

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4The event is called “informational” precisely because some traders in the market will receive private information on it.
sell arises.

To estimate our model, we will use a strategy proposed by Easley et al. (1997). They show how to use transaction data to estimate the parameters of the Glosten and Milgrom model with maximum likelihood. We will construct the likelihood function for the trading of an asset over many days. Our function takes into account that for some histories of trade agents herd. This means that in these histories, the probability of a buy or a sell order will be different from that when each agent follows his own private information. Our task will be more complicated than Easley et al.’s. In their set up, informed traders are perfectly informed on the value of the asset. Given that their signal is perfectly informative, these traders’ decisions will never be affected by the previous decisions and they will never herd. Therefore, the only thing that matters is the number of buys, sells and no trades during the day. The sequence in which orders arrive is irrelevant. In contrast, in our framework, history matters. Sequences with the same number of buys and sells may occur with different probabilities depending on the order in which buy and sell decisions arrive in the market. Therefore, we cannot limit our analysis to the number of orders but we have to consider the entire sequence of trades. Differently from Easley, Kiefer and O’Hara, we will estimate the model through a Bayesian analysis, i.e., we will start from some prior for the parameters and we will compute their posteriors conditional on the data.

We applied our methodology to the trading activity of Ashland Oil stock, a stock traded in the NYSE, in the first semester of 1994. We find that 9% of the trading periods are characterized by herd buy and 11% by herd sell. In 41 (out of 125) days of trade, herding periods were higher than 25% of the total periods of trade. Therefore, although, on average herding is not so common in the market, there are particular days in which it heavily characterizes traders’ behavior. In those days, herding behavior generated a significant deviation of the price path from the full information level. Through simulation results, we find that, because of herding, 4.3% of the times the price is more than 5% farther away from the price that would have prevailed if traders never herded.

The paper is organized as follows. Section 2 describes the theoretical model. Section 3 presents the likelihood function derived from the model. Section 4 describes the estimation strategy. Section 5 describes the data. Sections 6 presents the results, and Section 7 concludes. An Appendix contains all the proofs.
2 The Model

Our model is based on Glosten and Milgrom (1985) and on Easley and O’Hara (1992), who generalize Glosten and Milgrom to an economy where trading happens over many days.

In our economy there is one asset traded by a sequence of traders who interact with a market maker. Trade occurs over many trading days, indexed by \( d = 1, 2, 3, \ldots \). Time within each day is discrete and represented by a countably infinite set of trading dates indexed by \( t = 1, 2, 3, \ldots \).

The asset

The fundamental value of the asset in day \( d \), \( V_d \), is a random variable. Such a value does not change during the day. In day 1 the value \( V_1 \) is distributed on the support \( \{ v_L^1, v, v_H^1 \} \) with the following probabilities:

\[
\Pr(V_1 = v) = (1 - \alpha), \quad \Pr(V_1 = v_L^1) = \alpha(1 - \delta), \quad \text{and} \quad \Pr(V_1 = v_H^1) = \alpha\delta
\]

(with \( 0 < \alpha < 1 \) and \( 0 < \delta < 1 \)). In any future day \( d \geq 2 \), the asset value can remain the same as in the previous day, or change. In particular, \( V_d \) is equal to \( v_{d-1} \) with probability \( (1 - \alpha) \) and changes with probability \( \alpha \).\(^5\) In the latter case, it decreases to the value \( v_{d-1} - \Delta^L \) with probability \( \alpha(1 - \delta) \), and increases to \( v_{d-1} + \Delta^H \) with probability \( \alpha\delta \), where \( \Delta^L > 0 \) and \( \Delta^H > 0 \). Note that we are assuming that informational events are independent over days. To alleviate notation, we define \( v_d^H := v_{d-1} + \Delta^H \) and \( v_d^L := V_{d-1} - \Delta^L \). Finally, we assume that \( (1 - \delta)\Delta^L = \delta\Delta^H \) As will become clear in the next pages, we required such a condition for the price to be a martingale.

The market

The asset is exchanged in a specialist market. Its price is set by a competitive market maker (the specialist) who interacts with a sequence of traders. At any time \( t = 1, 2, 3, \ldots \) during the day a trader is randomly chosen to act and can buy, sell or decide not to trade. Each trade consists of the exchange of one unit of the asset for cash. The trader’s action space is, therefore, \( \mathcal{A} = \{ \text{buy}, \text{sell}, \text{no trade} \} \). We denote the action of the trader at time \( t \) in day \( d \) by \( X_t^d \). Moreover, we denote the history of trades until time \( t - 1 \) of day \( d \) by \( H_t^d \).

The market maker

\(^5\)Note that \( v_{d-1} \) is the realization of the random variable \( V_{d-1} \). We will use the convention of denoting random variables with capital letters and their realizations with lower case letters also in the following of the analysis. Furthermore, to simplify the notation, we write \( E(\cdot|y) \) to mean \( E(\cdot|Y = y) \), i.e., the expected value conditional on the realization \( y \) of the random variable \( Y \).
At any time $t$ of day $d$, the market maker sets the prices at which a trader can buy or sell the asset. When posting these prices, he must take into account the possibility of trading with traders who have some private information on the asset value. He will set different prices for buying and for selling, i.e., there will be a bid-ask spread. We denote the ask price (the price at which a trader can buy) at time $t$ by $a^d_t$ and the bid price (the price at which a trader can sell) by $b^d_t$.

Due to unmodeled potential competition, the market maker makes zero expected profits by setting the ask and bid prices equal to the expected value of the asset conditional on the information available at time $t$ and on the chosen action, i.e.,

$$a^d_t = E(V^d_t | h^d_t, X^d_t = \text{buy}, a^d_t, b^d_t),$$
$$b^d_t = E(V^d_t | h^d_t, X^d_t = \text{sell}, a^d_t, b^d_t).$$

We also define the “price” of the asset at time $t$ as the market maker’s expected value of the asset before the time-$t$ trader has traded: $p^d_t = E(V^d_t | h^d_t)$.

The traders

There are a countably infinite number of traders. Traders act in an exogenously determined sequential order. Each trader is chosen to take an action only once. Traders are of two types, informed and noise. The trader’s type is not publicly know, i.e., it is his private information.

Noise traders trade for unmodeled (e.g., liquidity) reasons: they buy with probability $\varepsilon$, sell with probability $\frac{1}{2} - \varepsilon$ and do not trade with probability $(1 - \varepsilon)$ (with $0 < \varepsilon < 1$).

Informed traders have private information on the asset value. They are present in the market in a day $d$ only if an event occurred at the beginning of the day that made the asset value go up or down with respect to the previous day. Informed traders receive a private binary signal on the new asset value and maximize their expected profit based on that signal (i.e., they are risk neutral). The signal is a random variable $S^d_t$ distributed on \{s^L, s^H\}. We denote the conditional probability function of $S^d_t$ given a realization $v^d_t$ of $V^d_t$ by $\sigma(s^d_t | v^d_t)$. We assume that, conditional on the asset value $v^d_t$, the random variables $S^d_t$ are independent and identically distributed across time. In particular, we assume that

$$\sigma(s^L | v^d_t) = \sigma(s^H | v^d_t) = q \in (0.5, 1).$$
An informed agent knows that an event has occurred and his signal is informative on whether the event is good or bad. Nevertheless, he is not completely sure of the effect of the event on the asset value. For instance, he knows that there has been a change in the investment strategy of a company, but cannot be completely sure that this change will affect the asset value in a positive or negative way. The precision of the signal \( q \) can also be interpreted as measuring the ability of traders to process the private information that they receive.

When no event occurs, there is nothing agents can learn in that day and, therefore, all agents in the market are noise. In contrast, when there is an event, some agents receive a private signal on the new asset value and go to the market to exploit it. In this case, the proportion of informed traders is \( \mu \in (0, 1) \). At each time \( t \) in an informed day, an informed trader is chosen with probability \( \mu \) and a noise trader with probability \( 1 - \mu \).

In addition to his signal, a trader at time \( t \) observes the history of trades and prices and the current price. Therefore, his expected value of the asset is

\[
E(V_d|h_t^d, s_t^d). 
\]

His payoff function \( U : \{v_d, v_{d-1}, v_d^H\} \times A \times [v_d^L, v_d^H]^2 \rightarrow \mathbb{R}^+ \) is defined as

\[
U(v_d, X_t^d, a_t^d, b_t^d) = \begin{cases} 
 v_d - a_t^d & \text{if } X_t^d = \text{buy}, \\
 0 & \text{if } X_t^d = \text{no trade}, \\
 b_t^d - v_d & \text{if } X_t^d = \text{sell}.
\end{cases} 
\]

The trader chooses \( X_t^d \) to maximize \( E(U(V_d, X_t^d, a_t^d, b_t^d)|h_t^d, s_t^d) \). Therefore, he finds it optimal to buy whenever \( E(V_d|h_t^d, s_t^d) \geq a_t^d \), and sell whenever \( E(V_d|h_t^d, s_t^d) \leq b_t^d \). He chooses not to trade when \( b_t^d < E(V_d|h_t^d, s_t^d) < a_t^d \).

### 2.1 Herd Behavior

Let us discuss now the predictions of our model. We start by considering the equilibrium prices.

**Proposition 1** At any time \( t \), there exists a unique bid and ask price for the asset, which satisfies \( b_t^d \leq p_t^d \leq a_t^d \).

The market maker takes into account that buying or selling orders contain private information and sets a spread between the price at which he is willing to sell and buy (Glosten and Milgrom, 1985). Equilibrium prices always exist because noise traders are willing to accept any loss and, therefore, the market will never shut down.
In order to discuss how herding arises in the financial market, let us introduce the formal definition of herd behavior.

**Definition 1** There is herd buying in equilibrium at time $t$ of day $d$ when, if an informed trader trades at that time, he buys with probability 1, i.e., $E(V_d|h_t^d, s_t^d) > a_t^d$, for any $s_t^d \in \{s_L, s_H\}$. There is herd selling in equilibrium at time $t$ of day $d$ when, if an informed trader trades at that time, he sells with probability 1, i.e., $E(V_d|h_t^d, s_t^d) < b_t^d$, for any $s_t^d \in \{s_L, s_H\}$.

**Definition 2** There is herd behavior in equilibrium between times $t'$ and $t''$ ($t'' > t'$) of day $d$ if, for $t = t', ..., t''$, there is either herd buying or herd selling.

Herding arises when informed traders act alike, choosing the same action independently of their private signal. Because of this, there is conformity of trading activity in the market.

Traders who herd can make the wrong decision. For instance, it is possible that in a trading day characterized by a positive informational event, traders in a period of herding neglect their positive information on the true asset value and decide to sell it. In the cases in which agents neglect the correct signal and take the opposite action, we say that herding is *misdirected*.

We can prove the following proposition:

**Proposition 2** During an informed trading day, herd behavior arises with positive probability. Furthermore, herd behavior can be misdirected.

Intuitively, the reason for the occurrence of herding is that the price, although efficiently set by the market maker, moves “too slowly.” When informed traders and market maker look at the past history of trades, they interpret it in different ways. Suppose that a sequence of buy orders arrives in the market. Informed traders, knowing that there has been an informational event, attach a certain probability to the fact that these orders come from previous traders with positive signals. The market maker attaches a lower probability to this event, as he has to take into account the possibility that there was no event at the beginning of the day and that all traders in the market trade for non-informational reasons. Therefore, after a sequence of buys, he will update the price up, but less than if he knew that an event had occurred for sure. As a result, even a trader with negative information can have an expected value of the asset higher than the ask price and, therefore,
can neglect his signal to herd buy. The same logic explains herd selling: after a sequence of sells, the bid price may be high enough that also traders with positive information find it optimal to sell. Avery and Zemsky (1998) have analyzed cases (see their IS2 and IS3 information setups) in which herd behavior can arise that are similar in spirit to that presented here. Our analysis differs from theirs in that they discuss herding in the context of theoretical examples that are not suitable for an empirical analysis.

The presence of herding in the market is, of course, important for the informational efficiency of prices. During periods of herd behavior, private information is not aggregated by the price. This happens because traders do not make use of the private information they have and, as a result, the price cannot aggregate such information.

While the price does not aggregate private information efficiently, even during a period of herding, the market maker does learn something on the true asset value. Indeed, even in a period of herding, he updates his belief on the fact that there has been an informational event or not, i.e., that the asset value has changed or not. Therefore, the bid and ask prices are updated even in a period of herding. To continue on the same example above, if the market maker observes more traders to buy the asset, he will give more and more weight to the fact that these traders are informed (liquidity traders would indeed buy or sell with the same probability). Hence, he will post higher prices. In contrast, the beliefs of the traders will not change, since they already know that an event has occurred, that they are in a herd and that the actions do not reflect private information. Because prices keep increasing and the traders’ beliefs don’t change, eventually herd behavior will disappear. Traders with negative signals will no longer find it optimal to neglect their signal and follow the herd, and private information will be again aggregated by the market price.

Essentially, while in our model there is herd behavior, there is no informational cascade. An informational cascade requires that the action be independent of the asset value. In a situation of informational cascade, the market maker is unable to infer anything on the asset value from the traders’ actions and, hence, is unable to update his prices. This is not the case in our model, since, while informed traders do not use their private signals, still the traders’ actions are informative on whether an event occurred or not.

\footnote{Formally, an informational cascade arises at time $t$ when $\Pr[X_t = x|h_t, a_t, b_t, v_t] = \Pr[X_t = x|h_t, a_t, b_t]$ for all $x \in A$ and for all $v_t$.}
Our model is a theory of temporary, intraday herd behavior. In our model, herd behavior arises because in some periods the prices, although efficiently set by the market maker, are such that traders have an incentive to buy (or to sell) independently of their private signals. Figure 1 shows the case in which, following a series of buy orders, buy herding arise. In the figure, we drew the bid and ask prices set by the market maker, the beliefs of a trader with a positive signal and those of a trader with a negative signal. After a long enough series of buys, the prices become lower than the expectations of the trader with a negative signal; when this happens herding arise.

During a period of herding, the informed traders do not update their beliefs at all. The market maker, instead, keeps updating his belief, since the trades change his posterior probability of whether an information event has occurred. Since the market maker gradually learns that an information event occurred, he will gradually start interpreting the history of past trades more and more similarly to the traders: as a result, herding will eventually stop. This is illustrated in Figure 2, where, after some more trades, the bid and ask prices cross the expectation of a trader with a negative signal. When this happens, traders with a negative signal start selling and the herd is broken. This result is formalized in the following proposition:

**Proposition 3** Suppose herd behavior starts at time $t$. Herd behavior cannot last forever, i.e., it stops with probability 1 at a time $t' \in (t, \infty)$.

Of course, during an informed day, herd behavior can start and break more than once, in different times of the day. Nevertheless, each period of herding has a limited life.

Given that information always flows to the market (even during time of herding behavior) and given that herding does not last forever, the price is able to aggregate the information that traders receive. Since this information is on average correct, the price will converge to the true asset value.\(^7\) We summarize this result in the next proposition:

**Proposition 4** In any day $d$ the asset price converges almost surely to the realized value of the asset.

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\(^7\)Recall that we have assumed that $(1 - \delta)\Delta^L_d = \delta\Delta^U_d$. This implies that $E[V_{d+1}|V_d] = V_d$. Since the price converges to the fundamental value almost surely, this guarantees that also the conditional expected price for day $d + 1$ be equal to the closing price in day $d$, i.e., it guarantees that the martingasle property of prices is satisfied.
Although information aggregation is slowed down during periods of herding, eventually the market maker learns whether in a day the market is characterized by a good or bad event or, on the contrary, is uninformed.

### 3 The Likelihood Function

One of the characteristic of the model is that it is possible to write the likelihood function for the sequence of buys, sells and no trades over many trading days. Let us define the complete history of trades in a single day as $h^d := h_{Td}^d$, where $T_d$ is the number of trading periods in day $d$. We denote the likelihood function by

$$
\mathcal{L}(\Psi) = \Pr \left( \{h^d\}_{d=1}^D | \Psi \right),
$$

where $\Psi = \{\alpha, \delta, q, \mu, \varepsilon\}$ is the vector of parameters that characterize our economy. In particular, let us recall that

- $\alpha$ is the probability that there is an information event in any given day;
- $\delta$ is the probability that the information event drives the fundamental value of the asset up;
- $q$ is the precision of the signal;
- $\mu$ is the probability that a trader is informed in an information day;
- $\varepsilon$ is the probability that an uninformed trader trades.

Note that we write the likelihood function as the conditional probability of the history of trades given the parameters of the model, and we omit the history of (bid and ask) prices. In our model there is no public information; for this reason there is a one-to-one mapping from trades to prices, and considering prices would be redundant. If we added public information to the model, the one-to-one mapping from trades to prices would break down, since prices would also reflect public information. Still, we could write the likelihood function in terms of trades only, since what really matters for our analysis is whether the belief of a trader is higher or lower than that of the market maker, and this is unaffected by public information.\(^8\)

In our model, Informational events at the beginning of each day are independent of each other. Trades in a day only depend on the value of the asset that day. For this reason, a particular history of trades over multiple

\(^8\)We discuss this point further at the end of this section.
days can be written as the product of the probability of the history of each single day. Therefore,
\[ \mathcal{L}(\Psi) = \Pr\left( \{h^d\}_{d=1}^{D} | \Psi \right) = \prod_{d=1}^{D} \Pr(h^d|\Psi). \]

Let us consider discuss now the likelihood of a history of trades in each single day. Note that
\[ \Pr(h^d|\Psi) = (1 - \alpha) \Pr(h^d|V_d = v_{d-1}, \Psi) + \alpha(1 - \delta) \Pr(h^d|V_d = v^L_d, \Psi) + \alpha \delta \Pr(h^d|V_d = v^H_d, \Psi). \]

This means that we need to describe the probability of a history of trades in any given day, conditional on that day being a day without an information event or with good or bad information event. Let us start from no information event days. During a non-informed day, the probability of a buy or sell is \( \frac{\varepsilon}{2} \), while the probability of a no trade is \( 1 - \varepsilon \). Let us denote by \( B_d, S_d \) and \( N_d \) the number of buys, sells and no trades in day \( d \). The probability of a history in day \( d \) can be written as:
\[ \Pr(h^d|\Psi, V_d = v_{d-1}) = K \left( \frac{\varepsilon}{2} \right)^{B_d+S_d} \left( 1 - \varepsilon \right)^{N_d}, \]
where \( K \) is the number of permutations of \( B_d \) buys, \( S_d \) sells and \( N_d \) no trades. From the above formula, it is clear that in a no-event day, we expect liquidity traders to buy or sell in a balanced way. In contrast, given that informed traders follow an informative signal, when there is an informational event there will be either a prevalence of buys or a prevalence of sells. This is the feature of the trading process that allows us to distinguish between information event day and no-information days.

Let us now compute the likelihood for a day in which there was a good information event. Since, in a day of information event, the probability of a trade at any time depends on the evolution of beliefs until then, the probability of a history of trades must be computed as
\[ \Pr(h^d_t|V_d = v^H_d) = \prod_{s=1}^{t} \Pr(x^d_s|h^d_s, V_d = v^H_d), \]
i.e., the probability of each trade depends on the previous history of trades.

Since, in an information day, the probability of a trade at time \( t \) depends on the sequence of trades until time \( t \), our likelihood function cannot be written as a simple function of the number of buys, sells and no trades in
each day, as done in Easley, Kiefer and O’Hara (1997). According to our model, the sequence of trades, not just the number of transactions, conveys information. Having many buy orders at the beginning of the day is not necessarily equivalent to having the same number of buy orders spread during the day. In fact, if there is a concentration of, for instance, buys at the beginning of the day, this may create herd behavior. The market maker in periods of herding will have to update his quotes (beliefs) in a different way than in the absence of herding. Furthermore, the probability of a particular sequence of trades in such a period of herding is different from the probability of the same sequence in the absence of herding. Therefore, to estimate our three parameters, we will not only use the number of buys, sells, and no trades. Rather, we will use the entire sequence of trades during each day.

In order to compute $\Pr(x^d_t|h^d_s,V_d = v^H_d)$, we need to distinguish those periods when agents follow their own signal from those periods in which they herd. Let us start from the case in which there is no herding (i.e., informed traders follow their own signal). In this case, the probability of observing a buy, a sell or a no trade at time $t$ in a positively informed day are

$$\Pr(\text{buy}^d_t|h^d_t,V_d = v^H_d) = \mu q + (1 - \mu)\epsilon,$$

$$\Pr(\text{sell}^d_t|h^d_t,V_d = v^H_d) = \mu (1 - q) + (1 - \mu)\epsilon,$$

$$\Pr(\text{nt}^d_t|h^d_t,V_d = v^H_d) = (1 - \mu)(1 - \epsilon).$$

As we illustrated in the previous section, after a prevalence of buys, the expectation of a trader with a negative signal can be higher than the equilibrium ask price. In such a case, an informed trader will buy independently of his signal. In this case, the probability of observing a buy, a sell or a no trade are

$$\Pr(\text{buy}^d_t|h^d_t,V_d = v^H_d) = \mu + (1 - \mu)\frac{\epsilon}{2},$$

$$\Pr(\text{sell}^d_t|h^d_t,V_d = v^H_d) = (1 - \mu)\frac{\epsilon}{2},$$

$$\Pr(\text{nt}^d_t|h^d_t,V_d = v^H_d) = (1 - \mu)(1 - \epsilon).$$

Similarly, after a prevalence of sells, the expectation of a trader with a positive signal can be lower than the bid price. In this case there will be herd selling and the probabilities of each action will be
\[
\begin{align*}
\Pr(buy^d_t|h^d_t, V_d = v^H_d) &= (1 - \mu) \frac{\varepsilon}{2}, \\
\Pr(sell^d_t|h^d_t, V_d = v^H_d) &= \mu + (1 - \mu) \frac{\varepsilon}{2}, \\
\Pr(nt^d_t|h^d_t, V_d = v^H_d) &= (1 - \mu)(1 - \varepsilon).
\end{align*}
\]

If we define three indicator functions \(I, I_{hb}, I_{hs}\) for each case above (i.e., \(I_{hb}\) takes value 1 if there is herd buy and 0 otherwise), we can write the probability of, for instance, a buy order in the case of a good event day as:

\[
\begin{align*}
\Pr(buy^d_t|h^d_t, V_d = v^H_d) &= \left[ \mu + (1 - \mu) \frac{\varepsilon}{2} \right] I_{hb} + \left[ (1 - \mu) \frac{\varepsilon}{2} \right] (I_{hs} + I_{ms}) + \left[ \mu q + (1 - \mu) \right] (I + I_{mb}).
\end{align*}
\]

We can write the probability of any other action conditional on a good event in a similar way. The analysis for the case of a bad event is identical.

---

\[9\text{Besides the normal case and those of herd buying and herd selling, there are two intermediate cases. The first occurs when, after a positive trade imbalance, } b^d_t < E(V|h^d_t, s^L_t) < p^d_t. \text{ In this case, in equilibrium, the market maker computes the bid assuming that only a noise trader would sell while an agent receiving a negative signal would not trade, and this is indeed what happens. In this instance, the probabilities of a trade in a good information day will be:}
\]

\[
\begin{align*}
\Pr(buy^d_t|h^d_t, V_d = v^H_d) &= \mu q + (1 - \mu) \frac{\varepsilon}{2}, \\
\Pr(sell^d_t|h^d_t, V_d = v^H_d) &= (1 - \mu) \frac{\varepsilon}{2}, \\
\Pr(nt^d_t|h^d_t, V_d = v^H_d) &= (1 - \mu)(1 - \varepsilon) + \mu q.
\end{align*}
\]

We define the indicator function \(I_{mb}\) taking value 1 if this case occurs and 0 otherwise. The other case occurs when, after a sequence of sells, \(p_t < E(V|h^d_t, s^H_t) < a^d_t\), and in equilibrium the market maker sets the ask assuming that only a noise trader would buy, as it is indeed the case. Therefore, in a good information day

\[
\begin{align*}
\Pr(buy^d_t|h^d_t, V_d = v^H_d) &= (1 - \mu) \frac{\varepsilon}{2}, \\
\Pr(sell^d_t|h^d_t, V_d = v^H_d) &= \mu(1 - q) + (1 - \mu) \frac{\varepsilon}{2}, \\
\Pr(nt^d_t|h^d_t, V_d = v^H_d) &= (1 - \mu)(1 - \varepsilon) + \mu q.
\end{align*}
\]

For this case we will use the indicator function \(I_{ms}\).

The probabilities of each action in the case of a bad information day are computed similarly.
To conclude our description, we need to explain how, for any values of the parameters, we can determine whether we are in a situation of no herd, herd buy or herd sell. Herd buying arises when an informed trader values the asset more than the ask price posted by the market maker, independently of his private information. Therefore, in order to check whether there is herd buying or herd selling at time $t$, we simply need to compare the expectation of the trader (conditional on each signal) with that of the market maker.

One could believe that in order to compare the traders’ and the market maker’s beliefs, and decide in which of the cases illustrated above we are at any time $t$, we would need data on the magnitude of the shock of the information event that buffets the asset’s fundamental (i.e., that we would need to estimate $\Delta^H$ and $\Delta^L$). We can easily show that this is not the case. The expected value of the asset for a trader at time $t$ is

$$E(V|d^t_i, s^t_i) = v^H_d \Pr(V_d = v^H_d|h^d_t, s^t_i) + v^L_d \Pr(V_d = v^L_d|h^d_t, s^t_i) = v_{d-1} - \Delta^L \Pr(V_d = v^L_d|h^d_t, s^t_i) + \Delta^H \Pr(V_d = v^H_d|h^d_t, s^t_i).$$

On the other hand, the expected value of the market maker is

$$E(V|h^d_t) = v^H_d \Pr(V_d = v^H_d|h^d_t) + v_{d-1} \Pr(V_d = v^L_d|h^d_t) + v^H_d \Pr(V_d = v^H_d|h^d_t) = -\Delta^L \Pr(V_d = v^L_d|h^d_t) + v_{d-1} + \Delta^H \Pr(V_d = v^H_d|h^d_t),$$

Therefore, the difference between the two expectations is

$$\begin{align*}
&\left[v_{d-1} - \Delta^L \Pr(V_d = v^L_d|h^d_t) + \Delta^H \Pr(V_d = v^H_d|h^d_t)\right] - \\
&\left[v_{d-1} - \Delta^L \Pr(V_d = v^L_d|h^d_t, s^t_i) + \Delta^H \Pr(V_d = v^H_d|h^d_t, s^t_i)\right] \\
= &\left[-\frac{1-\delta}{\delta} \Delta^H \Pr(V_d = v^H_d|h^d_t) + \Delta^H \Pr(V_d = v^H_d|h^d_t)\right] - \\
&\left[-\frac{1-\delta}{\delta} \Delta^H \Pr(V_d = v^H_d|h^d_t, s^t_i) + \Delta^H \Pr(V_d = v^H_d|h^d_t, s^t_i)\right] = \\
&\Delta^H \left[\frac{1-\delta}{\delta} \left(\Pr(V_d = v^L_d|h^d_t, s^t_i) - \Pr(V_d = v^L_d|h^d_t)\right) - \Pr(V_d = v^H_d|h^d_t, s^t_i) + \Pr(V_d = v^H_d|H^d_t)\right]
\end{align*}$$

whose sign is independent of how big the positive or negative news is.\footnote{For simplicity, we have studied the difference between the trader’s expectation and the asset price (i.e., the market maker’s expectation before trader $t$ trades). It is straightforward to repeat the argument for the bid and the ask prices.} This means that, in order to understand whether we are in a period of herding or...
not, we do not need to estimate the magnitude of the information shock that hits the asset.

4 Bayesian Estimation of the Model

The main objective of our work is to estimate the structural model of herd behavior that we have just illustrated. After estimating the model, we will be able to detect the herding periods during the trading days.

We carry out the estimation in a Bayesian framework, simulating the posterior distribution of the parameters conditional on the data. All the five parameters in the model are assumed to be random variables with a given prior distribution. For all the parameters, except $q$, the prior distribution is assumed to be uniform on the interval $[0, 1]$; the prior distribution of $q$ is assumed to be uniform in the interval $[1/2, 1]$.

In order to draw from the posterior distributions of the parameters, we generate random samples using a Markov-Chain Monte Carlo (MCMC) procedure. In particular, using the "Metropolis-Hasting" method, we sequentially sample from the posterior distribution of one parameter conditional on the other four.\(^{11}\)

Under regularity conditions (see Geweke, 1996 and 1997) here satisfied, the Markov chain so produced converges, and yields a sample from the joint posterior distribution of the parameters conditional on the data. In the Section “Results” we present the average and the standard deviation of 1,000,000 draws from the posterior distributions of each parameter.\(^{12}\) In computing the average, we discard the first 100,000 draws from a “burn in” phase.

Finally, let us remark that to estimate all our parameters we clearly need data on more than one trading day. The parameters $\alpha$ and $\delta$ define the probability that there is an event at the beginning of a trading day and that the event is good or bad. Therefore, if we used data on one day only, this would be equivalent to observing just one draw from a joint distribution with parameters $\alpha$ and $\delta$. Clearly we would be unable to estimate these two parameters. For this reason, we will use data for many days.

\(^{11}\)As starting values we use the mode of the posterior distribution (which, since the priors are flat, is the maximum of the likelihood function). We estimate the mode using a genetic algorithm optimization method.

\(^{12}\)The results we obtain are not significantly different if we use the posterior mode instead of the posterior mean.
5 Data

We use data for Ashland Oil, a stock traded in the New York Stock Exchange. We took data for this stock from the TAQ dataset. This dataset reports a complete list of the posted prices (quotes), the price at which the transactions occurred (trade), the size of the transactions and, of course, the time when the quotes were posted and the transaction occurred. We use transactions data on this stock for the first semester of 1994. In this period there were 125 trading days.

In order to extract the history of trades from this dataset, we had to make several transformations. First, these data do not say who initiated the trade, i.e., whether the transaction was a sell or a buy. In order to classify a trade as a sell or buy, we had to compare the trade with the quotes that were posted at the time of the transaction. For this purpose, we used the standard algorithm proposed by Lee and Ready (1991). We compared the transaction price with the quotes that were posted just before the transaction occurred. Any trade above the midpoint was classified as a buy and every trade below the midpoint was classified as a sell; trades at the midpoint were classified as a buy or a sell according to whether this price had increased or decreased with respect to the previous transaction price. Given that transaction prices are reported with a delay, we followed Lee and Ready (1991) suggestion of moving each quote ahead in time for five seconds.

Second, clearly these data do not contain any direct information on “no trades.” But, of course, there is a significant difference between an hour of trading in which many transactions occur and an hour in which there is none. We used the convention of classifying any period of two minutes in which no transaction occurred as a no trade. For instance, we considered a period of 20 minutes between two transactions as ten no trades. The choice of two minutes is, of course, arbitrary, as it would be for any other period. As a robustness check, we also used other, alternative, intervals.

As we said, we considered a sample period with 125 trading days. With our two minute rule, we had on average 208 decisions (either a buy, or a sell or a no-trade) a day; the maximum number of trades in a day was 301 and the minimum 142 (the standard deviation was 32). Our sample was fairly balanced: 26% of periods were buys, 26% were sells and 48% were no trades.

---

13 This is the same stock studied by Easley et al. (1997).
14 If there was no change, then we looked at the previous price movement and so on. As a matter of fact, there were only 16 trades at the midpoint in our dataset.
6 Results

Table 1 shows the mode and standard deviation of the posterior distribution of the model parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mode</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.7</td>
<td>0.06</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.58</td>
<td>0.10</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.20</td>
<td>0.02</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.45</td>
<td>0.01</td>
</tr>
<tr>
<td>$q$</td>
<td>0.68</td>
<td>0.03</td>
</tr>
</tbody>
</table>

If the specification we adopted is correct, informational events are quite frequent: 70% of trading days are in fact classified as days in which some trading activity was motivated by private information. There is a very small imbalance between good and bad news. Good informational events account for almost 58% of the informed days (although the parameter has a relatively high standard deviation). During informed days, the proportion of traders with private information is, on average, 20%. The remaining trading activity is explained by noise traders. Noise traders traded 45% of the time, and did not the remaining 55%. The precision of private information is just below 70%.

Such a precision of the signal (obviously lower than 1 given the standard deviation of the posterior distribution), opens the door to herd behavior. On the basis of these parameters, we tracked down the beliefs of the traders (with a positive or negative signal) and the beliefs of the market maker (i.e., the bid and ask prices) during each trading day. By comparing such beliefs we can detect periods in which, according to our model, there was herd behavior in the market. These are periods in which an informed trader would have made the same decision independently of the signal he received. For instance, when the belief of the trader is higher than the equilibrium ask even if he received a negative signal, we classify this period as herd buying. Similarly, when the belief of the trader is lower than the equilibrium bid even if he received a positive signal, we classify this period as herd selling. We found that 9% of trading periods are of herd buying and 11% are of herd selling. It is important to remark that herding periods are relevant for the informational (in)efficiency of the market. Indeed, during herding periods, the market is unable to learn whether the traders received a positive or a
negative signal. Although the market maker still learns something from the trading activity (namely, whether he is in an informed or uninformed day), information is aggregated less efficiently and the price converges more slowly to the fundamental asset value. For the period under analysis, we found that herding was pronounced in 54 days (out of 125). In such days, at least 10% of the trading periods were characterized by herding behavior. In 30 days, in particular, herding was very pronounced, since it characterized more than half of the trading periods.

Table 2: Number of days in which herding periods were frequent.

<table>
<thead>
<tr>
<th>&gt; 50%</th>
<th>&gt; 25%</th>
<th>&gt; 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>43</td>
<td>54</td>
</tr>
</tbody>
</table>

We also studied the average length of a herd. Herd buys lasted on average 26 trading periods (which corresponds to about 49 minutes); the longest herd buy in the sample lasted for 160 periods (300 minutes). Herd sells lasted on average for 33 periods, and the longest herd sell in the sample lasted 171 periods (321 minutes). There is, however, high variability in the length of herds (the standard deviation for herd buy and herd sell is 40 and 44). Finally herds tend to be concentrated in the middle of the day (with 208 trades per day on average, the mean buy-herd trade is the 124th and the mean sell-herd trade is the 133rd).

Figure 1 exemplifies what happens in a typical herd buy day. The black line represents the expectations of the market maker (to keep the picture simple we do not report the bid and ask prices); the dotted line represents the expectation of a trader with a low signal, whereas the dashed line represents the expectation of a trader with a high signal. All expectations have been re-scaled between 0 and 1. The line at the bottom represents the trade imbalance (measured on the right vertical axis), i.e., the difference between the number of buy and sell trades.

At the beginning of the day, a series of buy trades arrive at the market. Both traders and market maker update their expectations upwards. Since, however, the market maker does not know whether there are informed traders in the market, he updates his expectations by a smaller amount than the traders; after 31 trade-periods, the expectations of all informed traders are higher than that of the market make (no matter what signal they receive) and a herd-buy starts. During the herd-buy (from trade-period 31 to trade-period 67), traders do not update their expectations (since they realize...
that trades do not convey information on whether the information event was good or bad). In contrast, the expectation of the market maker changes: a series of buys between period 56 and 66 raises the probability that the market maker attaches to the market being informed. As a result, the market maker interprets the history of trades more similarly to the traders and his expectations cross those of a trader with a low signal. At trade period 67, the herd is broken.

A new herd starts at period 83 and lasts until period 148. During this time, the trade imbalance fluctuates up and down, thus convincing the market maker that the market is likely to be uninformed (his expectation converges towards the unconditional expected value of the asset). It is only when a series of consecutive buys arrive at the market that the market maker’s expectations rise again, breaking this second span of herding. Finally, at the end of the day, a series of buying arriving to the market make expectations of all market participants converge toward 1 (i.e., the market participants learns that there was a positive information event).

Until now we have analyzed how frequently herd behavior arises and for how long periods of herding last. A related important issue is to understand the extent to which herding behavior causes informational inefficiencies in the market. To have a better understanding of this, we simulated 10,000 days of trading in a financial market with our estimated parameters. We then simulated the same days of trading with the same parameters but assuming that traders, instead of behaving rationally as in our model, always followed private information. We took this as a benchmark case, since in this case all private information would be revealed by the trading decisions. We compared the price paths under the two scenarios. We considered the absolute difference at each time of every trading day between the simulated price and the full information price. We found that the average distance between the two prices is 0.4%. In other words, the presence of herding determined a deviation of the price from the full information level of 0.4% on average during each day. Moreover, in 4.3% of trading periods, the distance between the two prices was larger than 5%. This suggests that there are times when intraday herding affects the informational properties of the price in a very significant manner.
7 Conclusion

We estimated a model of herding behavior in financial markets. We used transaction data for a stock traded in the NYSE. We estimated the parameters of the structural model and detected periods in each trading day in which, according to our model, informed traders chose the same action independently of whether they had positive or negative private information on the value of the stock, i.e., they herded. We found that herding is present in the market. In some days of trade it is fairly pervasive. In our future research we will apply our methodology to verify whether herding is more pronounced in particular markets, and in particular times (like during financial crises).
References


8 Appendix

8.1 Proof of Proposition 1

First, we prove the existence of the ask price. Because of unmodeled potential Bertrand competition, the ask price at time \( t \), \( a_t^d \), must satisfy the condition

\[
a_t^d = E[V|\, h_t^d, X_t^d = 1, a_t^d, b_t^d].
\]

Let us define \( I_t^d \) as a random variable that takes value 0 if the agent at time \( t \) in day \( d \) is noise and 1 if he is informed. The expected value of the asset at time \( t \) in day \( d \), given a buy order at the ask price \( a_t^d \), is

\[
E[V|h_t^d, X_t^d = 1, a_t^d, b_t^d] = \Pr[I_t^d = 1|h_t^d, X_t^d = 1, a_t^d, b_t^d] + Pr[I_t^d = 0|a_t^d, b_t^d].
\]

Let us consider the correspondence \( \psi : [v_L^d, v_H^d] \Rightarrow [v_L^d, v_H^d] \) defined as \( \psi(a_t^d) := E[V|h_t^d, X_t^d = 1, a_t^d, b_t^d] \), and let us make the following observations:

1) If \( a_t^d > E[V|h_t^d, S_t^d = s_H^d], \Pr[I_t^d = 1|h_t^d, X_t^d = 1, a_t^d, b_t^d] = 0 \) and
\[ E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] = E[V|h^d_t, X^d_t = 1, I^d_t = 0]. \]

2) If \( E[V|h^d_t, S^d_t = s^L] < a^d_t < E[V|h^d_t, S^d_t = s^H] \), then \( E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] = E(V|h^d_t, I^d_t = 1, S^d_t = s^H, \) \( \Pr(I^d_t = 1, S^d_t = s^H| h^d_t) + E[V|h^d_t, I^d_t = 0] (1 - \Pr(I^d_t = 1, S^d_t = s^H| h^d_t)). \)

3) If \( a^d_t < E[V|h^d_t, S^d_t = s^L] \), then \( E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] = E(V|h^d_t, I^d_t = 1) \Pr(I^d_t = 1| h^d_t) + E(V|h^d_t, I^d_t = 0) (1 - \Pr(I^d_t = 1| h^d_t)), \)

where, of course, since the ask would be lower than the expectation of a trader with a negative signal, the conditional expected values and probabilities are computed assuming that an informed trader buys whatever signal he receives.

Finally note that

4) If \( a^d_t = E[V|h^d_t, S^d_t = s^H] \), then the informed trader receiving a positive signal can randomize between buying and not trading. If he buys with probability 0, then
\[ E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] \]

is equal to the expression indicated in Observation 1. If he buys with probability 1, then \( E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] \) is equal to the expression indicated in Observation 2. If he buys with any probability belonging to the interval \((0, 1)\), then \( E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] \) takes any value between these two expressions.

5) Similarly, if \( a^d_t = E[V|h^d_t, S^d_t = s^L] \), then the informed trader receiving a negative signal can randomize between buying and not trading. If he buys with probability 0, then
\[ E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] \]

is equal to the expression indicated in Observation 2. If he buys with probability 1, then \( E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] \) is equal to the expression indicated in Observation 3. If he buys with any probability belonging to the interval \((0, 1)\), then \( E[V|h^d_t, X^d_t = 1, a^d_t, b^d_t] \) takes any value between these two expressions.

Observations 1, 2, 3, 4, and 5 imply that the correspondence \( \psi \) is piecewise constant. Furthermore, for \( a^d_t = E[V|h^d_t, S^d_t = s^L] \) and \( a^d_t = E[V|h^d_t, S^d_t = s^H] \), \( \psi(a^d_t) \) takes all the values belonging to the intervals indicated above in observations 4 and 5. Therefore, it is immediate to see that the correspondence \( \psi(a^d_t) \) is non empty, convex-valued and has a closed graph. By Kakutani’s fixed point theorem, the correspondence has a fixed point. If
there is more than one fixed point, the ask price will be equal to the minimum of them (it is straightforward to show that the other fixed points do not represent an equilibrium, due to the potential Bertrand competition that the market maker faces).

The proof of the existence and uniqueness of the bid price is analogous.

The proof that \( b^d_t \leq p^d_t \leq a^d_t \) follows immediately from the proof in Glosten and Milgrom (1985, p. 81).

8.2 Proof of Proposition 2

Suppose \( V_d = V^L \). After a series of buy orders, the traders attach a higher probability to \( V_d = V^L \) (for whatever signal they receive). Such a probability goes to 1 when the number of buys goes to infinity. Therefore, we can always find a sufficiently high \( t' \) such that, after a history of \( t' \) buy orders, \( E[V_d|h^d_{t'+1}, s^L] > V_{d-1} + \epsilon \) where \( \epsilon > 0 \). Let us assume that in these \( t' \) periods herding has not arisen (otherwise the proposition is already proven). Now note that a no trade does not affect the traders’ beliefs, since it only comes from noise traders. That is, \( E[V_d|h^d_{t'+1}, nt_{t'+1}, s] = E[V_d|h^d_{t'+1}, nt_{t'+1}, s] \). In contrast, a no trade always increases the probability that the market maker attaches to \( V_d = V_{d-1} \). Indeed, after a no trade, \( \Pr[V_d = V_{d-1}|h^d_t, X^d_t = 0, a^d_t, b^d_t] = \frac{(1-\epsilon) Pr[V_d = V_{d-1}|h^d_t]}{(1-\mu)(1-\epsilon) Pr[V_d = V^H|h^d_t] + Pr[V_d = V^L|h^d_t] + (1-\epsilon) Pr[V_d = V_{d-1}|h^d_t]} \) which is obviously greater than \( \Pr[V_d = V_{d-1}|h^d_t] \).

Therefore, for any \( \epsilon > 0 \) and for any set of values for the parameters \( \{\alpha, \delta, \mu, q, \epsilon\} \), we can always find a number of \( t'' \) no trades sufficiently high that, in equilibrium, \( E[V_d|h^d_{t''+1}, X^d_{t''+1} = 1, a^d_{t''+1}, b^d_{t''+1}] < V_{d-1} + \epsilon. \) At this point, herd behavior arises and is misdirected. Such a history \( h^d_{t''+t''} \) occurs with positive probability, because of noise traders. This shows that (misdirected) herd buying occurs with positive probability. The proof for (misdirected) herd selling is analogous.

8.3 Proof of Proposition 3

Let us consider herd buying. During a situation of herd buying, informed traders buy independently of their signal. Suppose the period of herd buying is never broken, i.e., it lasts for ever after it has started at some time \( t \). In
such a case, the probability of each action after \( t \) is always the same. The conditional probabilities of a buy are given by \( \Pr[X_t^d = 1|h_t^d, a_t^d, b_t^d, V_d = V^H] = \Pr[X_t^d = 1|h_t^d, a_t^d, b_t^d, V_d = V^L] = [\mu + (1 - \mu)\frac{\beta}{2}] \).

\[ \Pr[X_t^d = 1|h_t^d, a_t^d, b_t^d, V_d = V_{d-1}] = \frac{\beta}{2}. \]

The probabilities of selling are \( \Pr[X_t^d = -1|h_t^d, a_t^d, b_t^d, V_d = V^H] = \Pr[X_t^d = -1|h_t^d, a_t^d, b_t^d, V_d = V^L] = (1 - \mu)\frac{\beta}{2}. \)

\[ \Pr[X_t^d = -1|h_t^d, a_t^d, b_t^d, V_d = V_{d-1}] = \frac{\beta}{2}. \]

Finally, the probabilities of not trading are \( \Pr[X_t^d = 0|h_t^d, a_t^d, b_t^d, V_d = V^H] = \Pr[X_t^d = 0|h_t^d, a_t^d, b_t^d, V_d = V^L] = (1 - \mu)(1 - \epsilon). \)

Let us denote by \( \beta, \sigma, \nu \), the number of buys, sells and no trades observed during a certain period after herding has started in \( t \). Then,

\[ \Pr[V_d = V^H|h_t^d, \beta, \sigma, \nu] = \frac{[\mu + (1 - \mu)\frac{\beta}{2}]^\beta[(1 - \mu)\frac{\beta}{2}]^\sigma[(1 - \mu)(1 - \epsilon)]^\nu}{K} \Pr[V_d = V^H|h_t^d], \]

\[ \Pr[V_d = V^L|h_t^d, \beta, \sigma, \nu] = \frac{[\mu + (1 - \mu)\frac{\beta}{2}]^\beta[(1 - \mu)\frac{\beta}{2}]^\sigma[(1 - \mu)(1 - \epsilon)]^\nu}{K} \Pr[V_d = V^L|h_t^d], \]

\[ \Pr[V_{d-1} = h_t^d, \beta, \sigma, \nu] = \left(\frac{2}{\sigma}\right)^{\beta + \sigma}(1 - \epsilon)^\nu \Pr[V_d = V_{d-1}|h_t^d]. \]

where \( K = [\mu + (1 - \mu)\frac{\beta}{2}]^\beta[(1 - \mu)\frac{\beta}{2}]^\sigma[(1 - \mu)(1 - \epsilon)]^\nu (\Pr[V_d = V^H|h_t^d] + \Pr[V_d = V^L|h_t^d]) + \right) \left(\frac{2}{\sigma}\right)^{\beta + \sigma}(1 - \epsilon)^\nu \Pr[V_d = V_{d-1}|h_t^d]. \]

Therefore, the probability that an event occurred at the beginning of day \( d \) is

\[ \Pr[V_d = V^H|h_t^d, \beta, \sigma, \nu] + \Pr[V_d = V^L|h_t^d, \beta, \sigma, \nu] = \frac{[\mu + (1 - \mu)\frac{\beta}{2}]^\beta[(1 - \mu)\frac{\beta}{2}]^\sigma[(1 - \mu)(1 - \epsilon)]^\nu(\Pr[V_d = V^H|h_t^d] + \Pr[V_d = V^L|h_t^d])}{K}. \]

The likelihood ratio between an event occurring or not is

\[ \frac{\Pr[V_d = V^H|h_t^d, \beta, \sigma, \nu] + \Pr[V_d = V^L|h_t^d, \beta, \sigma, \nu]}{\Pr[V_d = V_{d-1}|h_t^d, \beta, \sigma, \nu] = \frac{[\mu + (1 - \mu)\frac{\beta}{2}]^\beta[(1 - \mu)\frac{\beta}{2}]^\sigma[(1 - \mu)(1 - \epsilon)]^\nu(\Pr[V_d = V^H|h_t^d] + \Pr[V_d = V^L|h_t^d])}{K}. \]

The loglikelihood ratio can be expressed in the following way:

\[ \log \frac{[\mu + (1 - \mu)\frac{\beta}{2}]^\beta[(1 - \mu)\frac{\beta}{2}]^\sigma[(1 - \mu)(1 - \epsilon)]^\nu(\Pr[V_d = V^H|h_t^d] + \Pr[V_d = V^L|h_t^d])}{K} = \left(\frac{2}{\sigma}\right)^{\beta + \sigma}(1 - \epsilon)^\nu \Pr[V_d = V_{d-1}|h_t^d] \]

\[ \log \frac{\Pr[V_d = V^H|h_t^d] + \Pr[V_d = V^L|h_t^d]}{\Pr[V_d = V_{d-1}|h_t^d]} + \beta \log \frac{\mu + (1 - \mu)\frac{\beta}{2}}{\frac{\beta}{2}} + \sigma \log(1 - \mu) + \nu \log(1 - \mu). \]
By dividing both sides by the total number of buys, sells, and no trades, we obtain
\[
\frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V^H | h_s^d, \beta, \sigma, \nu] + \Pr[V_d = V^L | h_s^d, \beta, \sigma, \nu]}{\Pr[V_d = V_{d-1} | h_{t-1}^d, \beta, \sigma, \nu]} = \frac{\beta}{\beta + \sigma + \nu} \log \frac{\mu + (1-\mu) \frac{\epsilon}{2}}{\sigma + (1-\mu) \frac{\epsilon}{2}} + \beta \log(1-\mu) + \frac{\sigma}{\beta + \sigma + \nu} \log(1-\mu).
\]
Now, suppose there has been an event (i.e., \(V_d = V^L\) or \(V_d = V^H\)) and let \((\beta + \sigma + \nu) \to \infty\). Then,
\[
\frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V^H | h_s^d, \beta, \sigma, \nu] + \Pr[V_d = V^L | h_s^d, \beta, \sigma, \nu]}{\Pr[V_d = V_{d-1} | h_{t-1}^d, \beta, \sigma, \nu]} \to 0,
\]
\[
\frac{\beta}{\beta + \sigma + \nu} \to [\mu + (1-\mu) \frac{\epsilon}{2}],
\]
\[
\frac{\sigma}{\beta + \sigma + \nu} \to [(1-\mu) (1-\epsilon)],
\]
where the convergence almost surely just results from the law of large numbers. Hence, as time goes to infinity (and herd never stops):
\[
\frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V^H | h_s^d, \beta, \sigma, \nu] + \Pr[V_d = V^L | h_s^d, \beta, \sigma, \nu]}{\Pr[V_d = V_{d-1} | h_{t-1}^d, \beta, \sigma, \nu]} \to \log \frac{\mu + (1-\mu) \frac{\epsilon}{2}}{\sigma + (1-\mu) \frac{\epsilon}{2}} + (1-\mu)(1-\epsilon) \log(1-\mu) = [\mu + (1-\mu) \frac{\epsilon}{2}] \log \frac{\mu + (1-\mu) \frac{\epsilon}{2}}{\sigma + (1-\mu) \frac{\epsilon}{2}} + (1-\mu)(1-\epsilon) \log(1-\mu).
\]
It is easy to show that the RHS is positive. Therefore,
\[
\frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V^H | h_s^d, \beta, \sigma, \nu] + \Pr[V_d = V^L | h_s^d, \beta, \sigma, \nu]}{\Pr[V_d = V_{d-1} | h_{t-1}^d, \beta, \sigma, \nu]} \text{ converges to a positive constant.}
\]
This implies that
\[
\log \frac{\Pr[V_d = V^H | h_s^d, \beta, \sigma, \nu] + \Pr[V_d = V^L | h_s^d, \beta, \sigma, \nu]}{\Pr[V_d = V_{d-1} | h_{t-1}^d, \beta, \sigma, \nu]} \to +\infty,
\]
that is,
\[
\Pr[V_d = V^H | h_s^d, \beta, \sigma, \nu] + \Pr[V_d = V^L | h_s^d, \beta, \sigma, \nu] \to +\infty.
\]
If herd buys keeps forever, the belief that there has been no event converges to 0. During the period of herding, the informed traders do not update their beliefs, i.e., \(\Pr[V_d = V^H | h_s^d, \beta, \sigma, \nu] = \Pr[V_d = V^H | h_s^d, s] = \{s^L, s^H\}\). Hence, when \(\Pr[V_d = V_{d-1} | h_t^d, \beta, \sigma, \nu] \to 0\), \(E[V_d | h_t^d, \beta, \sigma, \nu, s^L] < E[V_d | h_t^d, \beta, \sigma, \nu, s^H]\), which contradicts that herd buying keeps forever.

The proof for the case of herd selling is analogous.
8.4 Proof of Proposition 4

Let us consider the case in which \( V_d = V^H \). We know from the proof of Proposition 3 that during periods of herding the market maker only learns about the probability of an informational event, while the loglikelihood ratio between the event being good or bad does not change. Indeed, from the proof of Proposition 3 we immediately obtain that, during periods herding,

\[
\frac{\Pr[V_d = V^H \mid h^d_t, \beta, \sigma, \nu]}{\Pr[V_d = V^L \mid h^d_t, \beta, \sigma, \nu]} = \frac{\Pr[V_d = V^H \mid h^d_t]}{\Pr[V_d = V^L \mid h^d_t]}.
\]

Now we show that during the (infinitely many) periods of non herding, \( \frac{\Pr[V_d = V^H \mid h^d_t]}{\Pr[V_d = V^L \mid h^d_t]} \to \infty \).

Suppose that during periods of non herding there are \( \beta \) buys, \( \sigma \) sells and \( \nu \) no trades. In such periods

\[
\Pr[V_d = V^H \mid h^d_t, \beta, \sigma, \nu] = \frac{[\mu(1-q) + (1-\mu) \frac{\nu}{2}]^\nu [(1-\mu)(1-\epsilon)]^\nu \Pr[V_d = V^H \mid h^d_t]}{K},
\]

\[
\Pr[V_d = V^L \mid h^d_t, \beta, \sigma, \nu] = \frac{[\mu(1-q) + (1-\mu) \frac{\nu}{2}]^\nu [(1-\mu)(1-\epsilon)]^\nu \Pr[V_d = V^L \mid h^d_t]}{K},
\]

where \( K = [\mu q + (1-\mu \frac{\nu}{2})]^{[\beta] \beta_1 \sigma} [\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{[\beta] \beta_2 \sigma} [(1-\mu)(1-\epsilon)]^\sigma \Pr[V_d = V^H \mid h^d_t] + [\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{[\beta] \beta_1 \sigma} [(1-\mu)(1-\epsilon)]^\sigma \Pr[V_d = V^L \mid h^d_t] + (\frac{\nu}{2})^{[\beta] \beta_1 + \sigma} (1-\epsilon)^\nu \Pr[V_d = V_{d-1} \mid h^d_t].
\]

The likelihood ratio between an event being good or bad is

\[
\frac{\Pr[V_d = V^H \mid h^d_t, \beta, \sigma, \nu]}{\Pr[V_d = V^L \mid h^d_t, \beta, \sigma, \nu]} = \frac{[\mu q + (1-\mu \frac{\nu}{2})]^{\beta_1 \sigma} [\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{\beta_2 \sigma} [(1-\mu)(1-\epsilon)]^\sigma \Pr[V_d = V^H \mid h^d_t]}{[\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{\beta_1 \sigma} [(1-\mu)(1-\epsilon)]^\sigma \Pr[V_d = V^L \mid h^d_t]}.
\]

The loglikelihood ratio can be expressed in the following way:

\[
\log \frac{[\mu q + (1-\mu \frac{\nu}{2})]^{\beta_1 \sigma} [\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{\beta_2 \sigma} [(1-\mu)(1-\epsilon)]^\sigma \Pr[V_d = V^H \mid h^d_t]}{[\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{\beta_1 \sigma} [(1-\mu)(1-\epsilon)]^\sigma \Pr[V_d = V^L \mid h^d_t]} = \log \frac{\Pr[V_d = V^H \mid h^d_t, \beta, \sigma, \nu]}{\Pr[V_d = V^L \mid h^d_t, \beta, \sigma, \nu]} + \beta \log \frac{[\mu q + (1-\mu \frac{\nu}{2})]^{\beta_1 \sigma}}{[\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{\beta_1 \sigma}} + \sigma \log \frac{[\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{\beta_2 \sigma}}{[\mu (1-q) + (1-\mu) \frac{\nu}{2}]^{\beta_2 \sigma}}.
\]

By dividing both sides by the total number of observed buys, sells and no trades, we obtain

\[
\frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V^H \mid h^d_t, \beta, \sigma, \nu]}{\Pr[V_d = V^L \mid h^d_t, \beta, \sigma, \nu]} = \frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V^H \mid h^d_t, \beta, \sigma, \nu]}{\Pr[V_d = V^L \mid h^d_t, \beta, \sigma, \nu]} + \frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V^H \mid h^d_t, \beta, \sigma, \nu]}{\Pr[V_d = V^L \mid h^d_t, \beta, \sigma, \nu]}.
\]
$$\frac{\beta}{\beta + \sigma + \nu} \log \frac{[\mu q + (1 - \mu) \frac{\varepsilon}{2}]^\alpha}{[\mu (1-q) + (1-\mu) \frac{\varepsilon}{2}]^\alpha} + \frac{\sigma}{\beta + \sigma + \nu} \log \frac{[\mu (1-q) + (1-\mu) \frac{\varepsilon}{2}]^\alpha}{[\mu q + (1-\mu) \frac{\varepsilon}{2}]^\alpha}.$$  

Let $$(\beta + \sigma + \nu) \to \infty$$. Then,

$$\frac{1}{\beta + \sigma + \nu} \log \left( \frac{\Pr[V_d = V_H | h_t^d] + \Pr[V_d = V_L | h_t^d]}{\Pr[V_d = V_{d-1} | h_t^d]} \right) \to 0,$$

$$\frac{\beta}{\beta + \sigma + \nu} \text{ a.s.} [\mu q + (1 - \mu) \frac{\varepsilon}{2}],$$

$$\frac{\sigma}{\beta + \sigma + \nu} \text{ a.s.} [\mu (1 - q) + (1 - \mu) \frac{\varepsilon}{2}],$$

$$\frac{\nu}{\beta + \sigma + \nu} \text{ a.s.} [(1 - \mu)(1 - \varepsilon)],$$

where the convergence almost surely just results from the law of large numbers. Hence, during the infinitely many periods of non-herding:

$$\frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V_H | h_t^d, \beta, \sigma, \nu]}{\Pr[V_d = V_L | h_t^d, \beta, \sigma, \nu]} \to$$

$$[\mu q + (1 - \mu) \frac{\varepsilon}{2}] \log \frac{[\mu q + (1-\mu) \frac{\varepsilon}{2}]^\alpha}{[\mu (1-q) + (1-\mu) \frac{\varepsilon}{2}]^\alpha} + [\mu (1-q) + (1-\mu) \frac{\varepsilon}{2}] \log \frac{[\mu (1-q) + (1-\mu) \frac{\varepsilon}{2}]^\alpha}{[\mu q + (1-\mu) \frac{\varepsilon}{2}]^\alpha}.$$  

It is easy to show that the RHS is positive. Therefore,

$$\frac{1}{\beta + \sigma + \nu} \log \frac{\Pr[V_d = V_H | h_t^d, \beta, \sigma, \nu]}{\Pr[V_d = V_L | h_t^d, \beta, \sigma, \nu]} \text{ converges to a positive constant, i.e., } \Pr[V_d = V_L | h_t^d, \beta, \sigma, \nu] \to 0.$$  

Since during periods of herding $\Pr[V_d = V_L | h_t^d, \beta, \sigma, \nu]$ remains constant, this result immediately shows that the market maker learns that the asset value cannot be $V_d = V_L$. An analogous proof shows that, during periods of non-herding, $\Pr[V_d = V_{d-1} | h_t^d, \beta, \sigma, \nu] \to 0$. From the proof of Proposition x we know that, if there were infinitely many times of herding, $\Pr[V_d = V_{d-1} | h_t^d, \beta, \sigma, \nu] \to 0$. Therefore, this immediately implies that the market maker learns that the asset value cannot be $V_d = V_{d-1}$. The proof of convergence for the case in which $V_d = V_L$ is analogous. The converges for the case in which $V_d = V_{d-1}$ is proven by invoking the law of large numbers, as done above (since the probability of each action is the same at each time $t$).
Figure 1: A Day of Buy Herding

Market maker’s expectation

Expectation of traders with high signal

Expectation of traders with high signal.

Trade imbalance